Optimal Joint Bond Design*

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Abstract

We study the optimal design of a joint borrowing arrangement among countries. In our model, a safe country, which has full commitment and never defaults, and a risky country, which lacks commitment and may default, participate in a joint borrowing scheme through which they allocate a predetermined amount of their bond issuance to a joint bond, which may earn a non-pecuniary premium. The joint borrowing scheme is flexible, and highlights the differences between pooled issuance, in which countries share the funds raised through the joint bond, and joint liability, in which one country guarantees the obligations of another one. We develop a simple but general condition that determines whether issuing a joint bond is welfare improving: if the total marginal increase in the amount raised by the countries – holding constant their borrowing decisions – is greater that the value of the joint liabilities that are originated, it is optimal to issue a positive amount of joint bond. We further decompose the welfare effects of varying the size of the joint bond into several distinct channels. We provide a quantitative analysis of joint borrowing agreements and find that Pareto improvements are possible.

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1 Introduction

The European sovereign debt crisis, whose origins date to the end of 2009 and whose effects are still felt today, changed the outlook of sovereign debt markets in Europe. As shown in Figure 5, starting in 2009, the interest rate spreads paid by most European countries widened to unprecedented levels since the creation of the Eurozone. The inability of several Eurozone member states to repay or refinance their government debt called for the introduction of mechanisms of mutual support among countries. Several stability funds that were put in place, like the European Financial Stability Facility (EFSF) and the European Financial Stabilization Mechanism (EFSM), were funded through joint borrowing schemes. But the policy proposal that caused the most heated debate in the public sphere was the creation of some form of permanent joint borrowing agreement among Eurozone countries.

A number of different proposals were openly endorsed by prominent economists and policy experts: Blue/Red Bonds, Eurobonds, Eurobills, ESBies, and Synthetic Bonds, among others, received widespread attention. The European Commission seriously studied the possibility of common issuance of sovereign bonds among the member states of the euro area. Despite the strong interest by policymakers and the general public, the formal analysis of this issue remains underdeveloped.

In this paper, we seek to provide a systematic treatment of the positive and normative consequences of joint borrowing agreements between sovereigns. In particular, we focus on characterizing under which conditions a joint borrowing scheme is desirable from a welfare standpoint. To carry out this task, we develop a framework in which a safe country, which never defaults on its debt, and a risky country, which may default, have the ability to enter into a joint borrowing scheme. Within the scheme, both countries allocate a fixed amount of their total bond issuance to a joint bond, while the rest of their debt remains individually issued by the countries.

We consider a class flexible schemes that allow us to separate the role of pooled issuance, through which countries split ex-ante the funds raised by the issuance of the joint bond, from the role of joint liability, through which one country guarantees ex-post the obligations of another country. Our framework allows for bonds issued by different countries, as well as the joint bond, to be priced by investors with different pricing kernels. This assumption allows us to account for non-pecuniary frictions present in international lending markets, like a safety premium, which have played an important role in policy discussions.

Because our analysis is motivated by the European experience, it is natural to study an environment with asymmetric countries. Incidentally, the asymmetry on commitment technologies simplifies the solution of the model. All the effects described in this paper also emerge when both countries lack commitment.

Our first theoretical result provides a simple test to determine whether the introduction of a joint borrowing agreement is socially beneficial. If the total marginal increase in the amount raised by the countries – holding constant their borrowing decisions – is greater that the value of the joint liabilities that are originated, it is optimal to issue a joint bond. We expect this result to hold very generally.[1]

Further exploiting the structure of the model, we show that this condition can be expressed as a function of four first-order effects. We refer to the first two as the risk sharing effect, which captures how raising resources through a joint bond may channel resources towards countries in worse economic conditions and vice versa, and the joint liability effect, which captures the benefits from having ex-post guarantees. These two effects capture how changes in bond prices affect welfare by redistributing resources and can take on any sign. We refer to the last two effects as the default change effect, which captures the increase in borrowing capacity generated by avoiding default, and the frictional effect, which accounts for the potential welfare gain associated with earning a non-pecuniary premium through the joint bond. These last two effects are always positive in our model, justifying why it may be desirable to issue a positive amount of joint bond.

Our second main result characterizes the welfare effect of varying the total amount of joint bond issued, providing qualitative and quantitative insights to the question of what is the optimal level of joint bond issuance. Two new first-order effects emerge far from the limit in which the level of issuance in small. These are related to how the uninternalized effects of the choices by the risky country affect the welfare of the safe country. We refer to the first new effect as the free riding effect, which is caused by the fact that the risky country does not account for how an increase in borrowing hurts the safe country by making borrowing through the joint bond more expensive. We brand the other effect as the default spillover effect. Both effects capture how varying the level of joint bond issuance changes the probability of default, either directly or through increased borrowing by the risky country, which causes first order welfare gains/losses for the safe country, because the risky country does not internalize the effects of its default decision in the safe country. In general, we expect all new effects to be negative, reducing the desirability of having a joint bond if countries can borrow freely without any other form of intervention.

We further show that Pareto improvements are possible for low levels of pooled issuance. Two frictions open the possibility of finding Pareto improvements: direct default spillovers and non-pecuniary pricing.

The two key findings of our quantitative exercise are the following. First, joint bond issuance significantly affects interest rates and the borrowing behavior of both countries. For most joint bond parameters, interest rates fall for both countries, which allows them to increase their borrowing. Second, perhaps surprisingly, joint liability arrangement seem to provide the greatest welfare gains. Intuitively, providing joint liability guarantees directly increases bond

[1] For instance, although the baseline model features time separable expected utility, exogenous output, and risk neutral international investors, our characterization remains unchanged if we allow for more general utility specifications, production, or international investors with more general pricing kernels.

prices, allowing them to earn an even higher non-pecuniary premium, which is the ultimate source of welfare gains.

We extend the results in several dimensions. First, we characterize the solution of the model when the default of the risky country causes direct output losses for the safe country. In this case, default spillovers are also first order and matter to determine qualitatively the desirability of a joint arrangement. Second, we show that the risky country overborrows if it is allowed to make borrowing and default choices freely. We characterize the optimal corrective policy, which directly address the free-riding effects and the default spillover effect. Third, we analyze the effects of bond tranching again when safe tranches may earn non-pecuniary benefits. Finally, we discuss how alternative assumptions would affect our results, including the possibility of having rollover risk and the alternative benchmark model in which the safe country has access to complete markets.

Literature This paper builds upon the growing literature that studies sovereign borrowing and default, recently surveyed by Aguiar and Amador (2013) and Uribe and Schmitt-Grohé (2016). This work mainly focuses on the problem of a single small open economy. Only a handful of papers incorporate linkages among borrowers. Kim and Zhang (2012) is one exception that studies the problem of decentralized borrowers within a given economy who default in a centralized way. The free-riding problem that emerges in that context is similar to the one that appears in our paper. Arellano and Bai (2013), which model interlinkages across sovereign markets due to joint renegotiation with risk averse investors, is another exception. In their work, the feedback between the default decisions of countries through the behavior of their common investors may give rise to multiple equilibria. In a more abstract setup, Korinek (2014) provides conditions under which global cooperation among policymakers is beneficial.[2]

Our paper provides a framework to interpret the different policy proposals regarding joint borrowing agreements that emerged during the recent European debt crisis. The European Commission proposal of stability bonds, as well as the synthetic bond proposal by Beck, Uhlig and Wagner (2011) only involve pooling of individually issued bonds. The Blue Bond/Red Bonds proposal of Delpla and Von Weizsacker (2011) involves both pooling and tranching. In this proposal, all national sovereign debt up to 60% of GDP would be mutualized and would back the joint bond called Blue Bond, through joint and several guarantees. The residual sovereign debt (Red Bonds) would still be issued nationally, providing fiscal discipline. The Eurobills proposal of Hellwig and Philippon (2011) only involves pooling and focuses on short-term instruments. They argue that these maturities have the highest potential for earning a liquidity premium and can also market discipline through the need to rollover the debt. Other policy proposals revolve around the generic idea of joint bond issuance to reap the benefits of a liquidity and safety premia, like the European Safe Bonds (ESBies) proposal by

[2]We abstract from the possibility that countries borrowing behavior is distorted due to political frictions that may alleviated by implementing corrective policies, as in Hatchondo, Martinez and Roch (2015) or Halac and Yared (2015).

Brunnermeier and al. (2011), which does not involve joint guarantees and is targeted towards jointly improving banks' balance sheets positions and sovereign fiscal stability.

Despite the abundance of policy proposals, there is little formal research on this area. To our knowledge, only a few papers address elements of the problem of joint bond issuance. Hatchondo, Martinez and Kursat Onder (2014) quantify the welfare gains from introducing a limited amount of non-defaultable debt in a canonical sovereign default model. They find that the introduction of non-defaultable debt is welfare increasing. He, Krishnamurthy and Milbradt (2015) study the possibility of pooled issuance of sovereign debt in an environment with risk neutral countries, rollover risk, and imperfect common knowledge. They find that only a sufficiently large amount of pooled issuance can increase welfare. Tirole (2015) proposes joint bond issuance schemes as a method of cross-country solidarity in the case of default spillovers, which we discuss in Section 5. In an environment with secondary market retrading of sovereign debt with differential enforcement between sovereign and domestic debtholders, Broner et al. (2014) show that cross country transfers can be welfare improving by freeing up domestic resources for investment.

Beyond the work on sovereign debt, this paper also relates to the literature on the macroeconomic shortage of safe assets and financial frictions. Among others, Caballero and Farhi (2014), Krishnamurthy and Vissing-Jorgensen (2012), Gorton and Ordonez (2013) and Gourinchas and Jeanne (2012) highlight the mismatch between a high demand and a low supply for safe assets.[3] By allowing for country/bond-specific pricing kernels, we incorporate the possibility that some bonds may earn non-pecuniary returns and study how this possibility affects the optimal joint borrowing scheme.[4]

Outline We describe the environment and lay out its recursive formulation in Section 2. We conduct the theoretical welfare analysis in Section 3, which allows us to interpret our quantitative results in Section 4. We study several extensions in Section 5. We conclude in Section 6 and relegate all proofs and derivations to the appendix.

2 Model

We study the problem of two small open economies that borrow from competitive foreign creditors. These economies have the ability to design ex-ante a joint borrowing agreement

[3]See also Bianchi, Hatchondo and Martinez (2012) for a model in which the accumulation of reserves allows countries to self insure.

[4]There is also a sizable literature that studies how the interaction between sovereign and banking risk provides a rationale for intervention in sovereign debt markets. For instance, Bolton and Jeanne (2011) analyze how the level of bond issuance of one country impacts the financial fragility in financially integrated economies. Gennaioli, Martin and Rossi (2014) explicitly models endogenous sovereign default and bank default to characterize the interlinkages between the two. Weymuller (2013) shows that safe sovereign debt enables banks to create safe bank debt, generating a safety multiplier.

to issue sovereign debt.[5] We first analyze how such a joint borrowing agreement affects the behavior of each country and then study whether such arrangement is desirable.

2.1 Environment

Time is discrete and indexed by $t \in 0, 1, \ldots, \infty$. There are two countries, denoted by $i = \{S, R\}$. Country R, which we refer to as risky, lacks commitment and may default on its sovereign debt. Country S, which we refer to as safe, is fully committed to repay and never defaults.

Endowments There is a single consumption good, which serves as numeraire. Each country receives every period a random endowment y_{it} of the consumption good. Endowment risk is the only source of uncertainty. We assume that endowment shocks are Markov and allow them to be correlated across countries and over time.

Preferences Each country seeks to maximize the preferences of a risk averse representative agent with time separable expected utility and a rate of time preference $\beta_i \in (0, 1)$, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t u_i \left(c_{it}\right),$$

where c_{it} denotes consumption and the flow utility function $u_i\left(\cdot\right)$ is increasing, concave, and satisfies an Inada condition. Preferences may differ across countries.

Market structure Both countries issue one period non-contingent debt to international investors. We denote by b_{it} the total face value of the bonds issued by country i due at date t. As described below, a fraction of these bonds may be issued as part of a joint bond. We denote the unit bond price received by country i on the bonds issued at date t by q_{it}, which is a function in equilibrium of borrowing choices and endowment realizations. Both countries face non-binding No-Ponzi conditions. Hence, each country faces the following budget constraint in periods with access to financial markets

$$c_{it} = y_{it} - b_{it} + q_{it}\left(\cdot\right) b_{i,t+1}.$$

Borrowing choices are made sequentially without commitment.

Default The safe country never entertains the possibility of defaulting, so its individually issued debt is risk-free. A default by the risky country is punished with (temporary) exclusion from financial markets and a direct output loss $L\left(y_R\right)$, which captures various default costs. As in Arellano (2008) and Chatterjee and Eyigungor (2012), we assume that direct output losses increase with the endowment level.

[5] We rule out direct fiscal transfers among countries, which may not be feasible because of political considerations. The discussion in the broader context of fiscal and monetary integration within an economic union is outside of the scope of this paper.

After defaulting, the risky country stochastically regains access to financial markets at a constant rate α. For simplicity, we assume that the recovery rate for debt after default is zero. The default of the risky country does not cause direct output losses to the safe country – we allow for this possibility in Section 5.

Joint borrowing scheme Both countries participate in a joint borrowing scheme that works as follows. Every period, each country i must issue $\theta_i \overline{b}$ bonds to be pooled into a joint bond, where $\overline{b}$ denotes the face value of the joint bond and θ_i is the share of country i individual liabilities in the joint bond. The shares θ_i must add up to unity, so $\sum_i \theta_i = 1$. We denote the price of the joint bond by $\tilde{q}_t^J$. When $\overline{b} = 0$, both countries operate independently of each other.

Country i chooses the number of unit bonds $b_{i,t+1}$ to issue every period. Therefore, the residual $b_{i,t+1} - \theta_i \overline{b}$ bonds are individually issued by country i. We denote the price of the individually issued bonds by $\tilde{q}_{it}$. Formally,

$$\begin{aligned} \theta_i \overline{b} &\quad \text{(Pooled bonds)} \\ b_{i,t+1} - \theta_i \overline{b} &\quad \text{(Individually issued bonds)}, \end{aligned}$$

where we work under the assumption that $b_{i,t+1} > \theta_i \overline{b}$, $\forall t$. We'll guarantee that it is always optimal for both countries to issue debt above the joint bond limit in all calibrations by assuming that they have a strong desire to borrow.[6]

Countries commit to not know which particular bonds form part of the joint bond. Therefore, the risky country cannot selectively default on the pooled bonds and repay the individually issued ones and vice versa. This assumption is consistent with the underlying friction of lack of commitment ex-post. Importantly, our formulation of the joint borrowing agreement does not alter the commitment technologies of each country.[7] Furthermore, when the risky country is excluded from international markets after defaulting, there is no joint bond issuance.

We make two further assumptions regarding the joint borrowing scheme. First, country i receives at issuance a price q_{it}^P per unit bond pooled into the joint bond (where P stands for pooled issuance). This price q_{it}^P corresponds to a linear combination, with weights κ and $1-\kappa$, between a) the price of the joint bond and b) the sum of the price of the individually issued bond and the pricing wedge Ω_t, which captures the price difference between the joint bond and the appropriately weighted individually issued bonds, as we formally define below. This first assumption introduces the possibility of *pooled issuance*, so that both countries split ex-ante the funds raised by the issuance of the joint bond.

Second, when the risky country defaults, the safe country becomes liable for a fraction $\lambda \in [0, 1]$ of the bonds issued by the risky country that were pooled into the joint bond.

[6] We describe how to allow for countries that save in the online appendix.

[7] This logic prevents countries from tranching individually issued bonds. If countries could commit to grant seniority to some payments over others, they could also commit to repay.

This second assumption introduces the possibility of *joint liability*, so that the safe country guarantees ex-post the promises made by the risky country.

Formally, the unit price received by country i on its total issuance, denoted by q_{it}, can be written as

$$q_{it} = \phi_{it} q_{it}^{P} + (1 - \phi_{it})\, \tilde{q}_{it}, \tag{1}$$

where the fraction of bonds issued by country i that are pooled into the joint bond $\phi_{it} \in [0, 1]$ is given by

$$\phi_{it} = \frac{\theta_i \overline{b}}{b_{i,t+1}}. \tag{2}$$

The first component of Equation (1) corresponds to the price received through the joint bond issuance. The second component corresponds to the price received through the individual issuance. Formally, the unit price q_{it}^{P} received by country i on the fraction of bonds pooled into the joint bond is given by

$$q_{it}^{P} = \kappa \tilde{q}_t^{J} + (1 - \kappa)\left(\tilde{q}_{it} + \Omega_t\right), \tag{3}$$

where $\tilde{q}_t^{J}$ denotes the price of the joint bond and Ω_t denotes the *pricing wedge* of the joint bond, formally defined by

$$\Omega_t \equiv \tilde{q}_t^{J} - \sum_i \theta_i \tilde{q}_{it}. \tag{4}$$

The pricing wedge Ω_t corresponds to the difference between the price of the joint bond and the appropriately weighted sum of the prices of the individually issued bonds. By construction the pricing wedge can be directly recovered from market data. As it will become clear, the value of the pricing wedge will depend in equilibrium on the degree of joint liability and the possible differences in the pricing kernels used to price individual bonds and the joint bond. Note that the definition of Ω_t guarantees that all funds from the issuance of the joint bond are distributed to the countries $\sum_i \theta_i q_{it}^{P} = \tilde{q}_t^{J}$.[8]

We use the tilde notation for the price of the joint bond and the individually issued bonds to emphasize that $\tilde{q}_t^{J}$ and $\tilde{q}_{it}$ are the prices of actually traded securities, as described below. The unit prices q_{it} and q_{it}^{P} are constructs of the joint borrowing scheme.

Definition 1. (Joint borrowing scheme) A joint borrowing scheme is characterized by four parameters $\left\{\overline{b}, \theta_i, \kappa, \lambda\right\}$: the face value of the joint bond $\overline{b} \in [0, b_{\max}]$, the share of bonds issued

[8] We could adopt a more flexible definition for q_{it}^{P}, distributing the pricing wedge asymmetrically across countries and allowing for country specific pooled issuance parameters κ_i, so that Equation (3) becomes

$$q_{it}^{P} = \kappa_i \tilde{q}_t^{J} + (1 - \kappa_i)\left(\tilde{q}_{it} + \chi_i \Omega_t\right)$$

We describe this case in the online appendix. The more general formulation emphasizes that any joint borrowing scheme must decide how to distribute the pricing wedge by choosing χ_i, an often overlooked argument in policy discussions. The baseline model distributes the pricing wedge symmetrically by implicitly assuming that $\chi_i = 1$. One could even consider a situations in which different components of the pricing wedge – like the liability or pricing wedge, described below – may be distributed differently among the countries.

by each country in the joint bond $\theta_i \in [0, 1]$, the degree of pooled issuance $\kappa \in [0, 1]$, and the degree of joint liability $\lambda \in [0, 1]$.

The parameter κ in Equation (3) modulates the degree of pooled issuance. When $\kappa \to 1$, $q^P_{it} = \tilde{q}^J_t$ and every country receives the joint bond price on the bonds pooled. When $\kappa \to 0$, there is no pooled issuance at all, so countries effectively receive a price commensurate with their default risk, captured in $\tilde{q}_{it}$, augmented by the pricing wedge Ω_t. The parameter λ modulates the degree of joint liability. When $\lambda \to 1$, the joint bond is issued at the relevant risk-free rate, since it is fully guaranteed by country S. When $\lambda \to 0$, the joint bond is simply a claim to a combination of individually issued bonds. The parameter θ_i varies the degree of involvement of each country. It is important that θ_i does not depend on the borrowing behavior of both countries. The long run share of output of each country i, $\theta_i = \mathbb{E}[y_i]/\sum_i \mathbb{E}[y_i]$ is a natural choice.

We study the properties of the time-invariant joint borrowing scheme under commitment. We first study the optimal determination of $\bar{b}$ for given levels of θ_i, κ, and λ, and then analyze how varying θ_i, κ, and λ affects the results. We can interpret the welfare analysis of this paper as the Ramsey problem solved ex-ante under commitment by a supra-national authority without access to fiscal transfers. Unrestricted transfers can trivially recover the first-best in our model.

International investors Policy discussions regarding joint borrowing schemes among sovereigns often revolve around the fact that some bonds may earn a non-pecuniary return: these bonds may be used as collateral, or they may earn a safety premium, perhaps because they are easier to securitize or re-trade. To capture that possibility, we assume that country i can individually borrow from risk neutral and perfectly competitive investors who require a country-specific rate of return $1 + r_i$. We also assume that the joint bond is priced under a different risk-free rate $1 + r_J$.[9] With little loss of generality, we further restrict the relation between the different rates in Assumption 1.

Assumption 1. *[International risk-free rates] We assume that bonds issued by the safe country have a non-pecuniary benefit relative to those issued by the safe country, that is, $r_S \leq r_R$. We also assume that the joint bond shares the same pricing kernel as the bonds individually issued by the safe country, that is, $r_J = r_S \leq r_R$.*

We assume that default decisions are made before investors determine a new price schedule.[10] The small open economy assumption guarantees that the borrowing behavior of both countries does not affect international interest rates. As in many previous studies, and for simplicity, we assume that investors face 100% haircuts on any issued bonds in case of default.

[9] A long tradition in macroeconomics, e.g. Woodford (1990), studies optimal policy problems when financial securities enjoy nonpecuniary returns.

[10] This timing assumption eliminates multiplicity problems caused by rollover risk as in Cole and Kehoe (2000). We discuss how the alternative timing assumption in which the risky country can borrow first and immediately default affects our results in the online appendix.

Equilibrium definition We focus on recursive Markov equilibria that depend on debt choices and output realizations as the payoff relevant state variables.

Definition 2. *(Equilibrium)* A recursive Markov equilibrium for a given joint borrowing scheme parameters $\left\{\overline{b}, \theta_i, \kappa, \lambda\right\}$ consists of (i) value functions $v_R\left(b_R, y_R\right)$, $v_R^c\left(b_R, y_R\right)$, $v_R^d\left(y_R\right)$, $v_S\left(b, y\right)$, $v_S^c\left(b, y\right)$, and $v_S^d\left(b, y\right)$, (ii) policy functions for consumption, borrowing, and default for the risky country and for consumption and borrowing for the safe country, and (iii) bond price functions $q_R\left(b_R', y_R\right)$ and $q_S\left(b_R', y_R\right)$, such that, given bond price functions, the value functions and policy functions satisfy the Bellman Equations (5) to (9) described below, while international lenders competitively offer bond price functions making zero profit.[11]

2.2 Relation to policy proposals

Our formulation of the joint borrowing agreement explicitly differentiates between pooled issuance and joint liability. Pooled issuance allows countries to split the funds raised by issuing the joint bond. It is effectively equivalent to agreeing to a set of implicit transfers at issuance determined by the prices of the individually issued bonds and the joint bond. Joint liability corresponds to the promise of paying the debt of another country in case of default. It is equivalent to the promise of transferring resources to investors in case of default. Policy proposals implicitly entail pooled issuance, joint liability, or both, often without being explicit about it. This paper provides a framework to dissect along these dimensions the core attributes of the different proposals.

	$\lambda = 0$	$\lambda = 1$
$\kappa = 0$	Repackaging	Joint Liability (Government Guarantees)
$\kappa = 1$	Pooled Issuance (ESBies/Synthetic Bonds)	Both (Eurobonds, Blue/Red Bonds)

Table 1: Joint Bond configurations

Table 1 relates our parameters to available proposals. When $\kappa \to 0$ and $\lambda = 1$, countries borrow independently, but country S is liable for $\lambda \theta_R \overline{b}$ bonds issued by the risky country – this is similar to system of direct government guarantees. When $\kappa \to 1$ and $\lambda \to 1$, countries pool the funds of the joint bond at issuance and maintain the joint liability. These two formulations are closest to the original Eurobonds proposal and the Blue bond/Red bond proposal. When $\kappa \to 1$ and $\lambda = 0$, countries split the proceeds from the issuance of the joint bond, but there is no joint liability ex-post – this is similar to the ESBies proposal.[12] When $\kappa \to 0$ and $\lambda \to 0$, our

[11] Importantly, countries take as given the parameters of the joint borrowing scheme: there is scope to further understand the substantially more complex problem in which countries behave strategically with respect to the choice of $\overline{b}$ and other parameters by a supranational authority, in a Stackelberg fashion. This is a natural topic for future research.

[12] We abstract from tranching in this paper, which is an integral part of the ESBies proposal. If tranching allows the joint bond to earn a non-pecuniary premium, our formulation can be used to understand its consequences.

formulation allows the countries to repackage a fraction of their issuance as a distinct bond, which nonetheless may be traded at a different price than the individually issued ones.

2.3 Recursive formulation and equilibrium

We now pose the problem solved by each country in recursive form. By design, the problem solved by the risky country is almost identical to the canonical sovereign default problem studied in Arellano (2008) or Aguiar and Gopinath (2006). The problem solved by the safe country is close to the canonical individual income fluctuation problem, as described in Ljungqvist and Sargent (2004). Importantly, the safe country/risky country formulation allows us to solve both problems sequentially. First, we solve the problem of the risky country. Subsequently, we solve the problem of the safe country, given the behavior of the risky country.

Risky country

Using the block recursive structure of the model to limit the number of state variables to two, b_R and y_R, for any bond price function $q_R(\cdot)$, the value function of the risky country $v_R(\cdot)$ satisfies

$$v_R(b_R, y_R) = \max_{d_R} \left\{ (1 - d_R) v_R^c(b_R, y_R) + d_R v_R^d(y_R) \right\}, \tag{5}$$

where $d_R = 1$ in case of default and $d_R = 0$ otherwise. The debt and endowment realizations of the safe country are not state variables for the risky one.

The value of repaying for the risky country (the index c stands for continuation, the index d below stands for default) is given by

$$\begin{aligned} v_R^c(b_R, y_R) = \max_{c_R, b'_R} & \left\{ u_R(c_R) + \beta_R \mathbb{E}_{y'_R|y_R} \left[v_R(b'_R, y'_R) \right] \right\} \\ \text{subject to} \quad & c_R = y_R - b_R + q_R(b'_R, y_R) b'_R, \end{aligned} \tag{6}$$

where the bond price function $q_R(b'_R, y_R)$ is determined as described below.

The value of defaulting for the risky country is given by

$$v_R^d(y_R) = u_R(y_R - L(y_R)) + \beta_R \alpha \mathbb{E}_{y'_R|y_R} \left[v_R(0, y'_R) \right] + \beta_R (1 - \alpha) \mathbb{E}_{y'_R|y_R} \left[v_R^d(y'_R) \right],$$

where $L(\cdot)$ is defined in Equation (26). The solution to the problem of the risky country consists of optimal policies for borrowing $b'_R(b_R, y_R)$, default $d(b_R, y_R)$, and consumption $c(b_R, y_R)$.

Safe country

Let $y = (y_R, y_S)$ denote the vector of endowments. Let $b = (b_R, b_S)$ and $b' = (b'_R, b'_S)$ denote the vectors of current and next period levels of debt. For any bond price function $q_S(\cdot)$, which depends nontrivially on (b_R, y_R), and default policy of the risky country $d(\cdot)$, the value function of the safe country $v_S(\cdot)$ satisfies

$$v_S(b,y) = (1 - d_R(b_R, y_R))\, v_S^c(b,y) + d_R(b_R, y_R)\, v_S^d\left(b, y^{dd}\right), \tag{7}$$

where the vector $y^{dd} = \left(y_R, y_S - \lambda\theta^R\overline{b}\right)$ accounts for the fact that the safe country faces an additional liability $\lambda\theta^R\overline{b}$ over the fraction λ of bonds defaulted by the risky country that belong to the joint bond. The value function of the safe country when the risky one repays is given by[13]

$$v_S^c(b,y) = \max_{c_S, b_S'} \left\{ u_S(c_S) + \beta_S \mathbb{E}_{y'|y}\left[v_S(b', y')\right] \right\} \tag{8}$$
$$\text{subject to} \quad c_S = y_S - b_S + q_S(b_R', y_R)\, b_S',$$

where the bond price function $q_S(b_R', y_R)$ is determined as described below. Note that the price faced by the safe country is a function of the endowment and debt choices of the safe country, through the joint bond. The value function of the safe country when the risky country defaults is given by

$$v_S^d(b,y) = \max_{c_S, b_S'} \left\{ u_S(c_S) + \beta_S \alpha \mathbb{E}_{y'|y}\left[v_S(b', y')\right] + \beta_S(1-\alpha)\,\mathbb{E}_{y'|y}\left[v_S^d(b', y')\right] \right\} \tag{9}$$
$$\text{subject to} \quad c_S = y_S - b_S + \tilde{q}_S b_S'.$$

Note that, when the risky country is excluded from financial markets, the risky country borrows at the constant risk-free rate $\tilde{q}_S$.

Equilibrium bond pricing

Because the safe country has full commitment, it borrows individually at the risk-free rate offered by the international investors. Formally,

$$\tilde{q}_S = \frac{1}{1 + r_S}. \tag{10}$$

Bonds individually issued by the risky country entertain a risk premium. In equilibrium, international investors anticipate the default behavior of country R. Hence, the bond price

[13] If country S decided to guarantee a fraction λ of *all* the debt issued by country R, not only the one in the joint bond, y^{dd} becomes $\{y_R, y_S - b_R\}$. We do not consider that possibility and exclusively focus on joint liability through the pooled bond.

function faced by country R on its individually issued bonds must satisfy

$$\tilde{q}_R\left(b'_R, y_R\right) = \frac{1 - \mathbb{E}_{y'_R|y_R}\left[d\left(b'_R, y'_R\right)\right]}{1 + r_R}. \tag{11}$$

Using Assumption 1, we can express the joint bond price function as

$$\tilde{q}^J\left(b'_R, y_R\right) = \theta_S \tilde{q}_S + \theta_R\left(\lambda \frac{1}{1 + r_J} + (1 - \lambda)\frac{1 + r_R}{1 + r_J}\tilde{q}_R\left(b'_R, y_R\right)\right). \tag{12}$$

The price of the joint bond is exclusively a function of the relevant state variables for the risky country. The first component corresponds to the present value of the payments derived from the bonds pooled by the safe country. The second component corresponds to the present value of the payments made by the risky country as well as those coming from the joint liability. The degree of joint liability matters to determine $\tilde{q}^J(\cdot)$, but not the degree of joint issuance.

Once the prices of both individually issued bonds and the joint bond are determined, one can use Equations (1) to (4) to determine the equilibrium country specific bond price functions q_i and q_i^P, the fraction of bonds issued by country i that and the pricing wedge Ω. Exploiting the equilibrium joint bond price function in Equation (12), we can express the equilibrium pricing wedge $\Omega = \tilde{q}^J - \sum \theta_i \tilde{q}_i$ as follows

$$\Omega\left(b'_R, y_R\right) = \underbrace{\theta_R \frac{r_R - r_J}{1 + r_J}\tilde{q}_R\left(b'_R, y_R\right)}_{\Omega^F \text{ (Frictional Wedge)}} + \underbrace{\lambda\theta_R\left(\frac{1}{1 + r_J} - \frac{1 + r_R}{1 + r_J}\tilde{q}_R\left(b'_R, y_R\right)\right)}_{\Omega^L \text{ (Liability Wedge)}}. \tag{13}$$

The pricing wedge has two components. The first one captures the increase in the value of issuance generated by issuing the joint bond in a market with a more favorable pricing kernel – we refer to it as the *frictional wedge*. It can be understood as a form of seigniorage earned by issuing the joint bond. The second component captures the increase in funds raised at issuance due to the additional liability assumed by country S – we refer to it as the *liability wedge*. The liability wedge can also be expressed as

$$\Omega^L\left(b'_R, y_R\right) = \lambda\theta_R \frac{\mathbb{E}_{y'_R|y_R}\left[d\left(b'_R, y'_R\right)\right]}{1 + r_J}$$

Note that both the frictional and the liability wedges are positive, $\Omega^F, \Omega^L \geq 0$, and that the pricing wedge $\Omega\left(b'_R, y_R\right)$ and the joint bond price $\tilde{q}^J\left(b'_R, y_R\right)$ do not depend on the behavior of the safe country.

Figure 1 illustrates the behavior of the bond pricing functions for our benchmark calibration, described below, for a joint borrowing scheme with values $\overline{b} = 0.1$, $\theta_R = 0.5$, and $\lambda = \kappa = 0.5$. For that particular parametrization the safe country receives a lower unit bond price when borrowing more. This occurs because the pricing wedge is large enough that

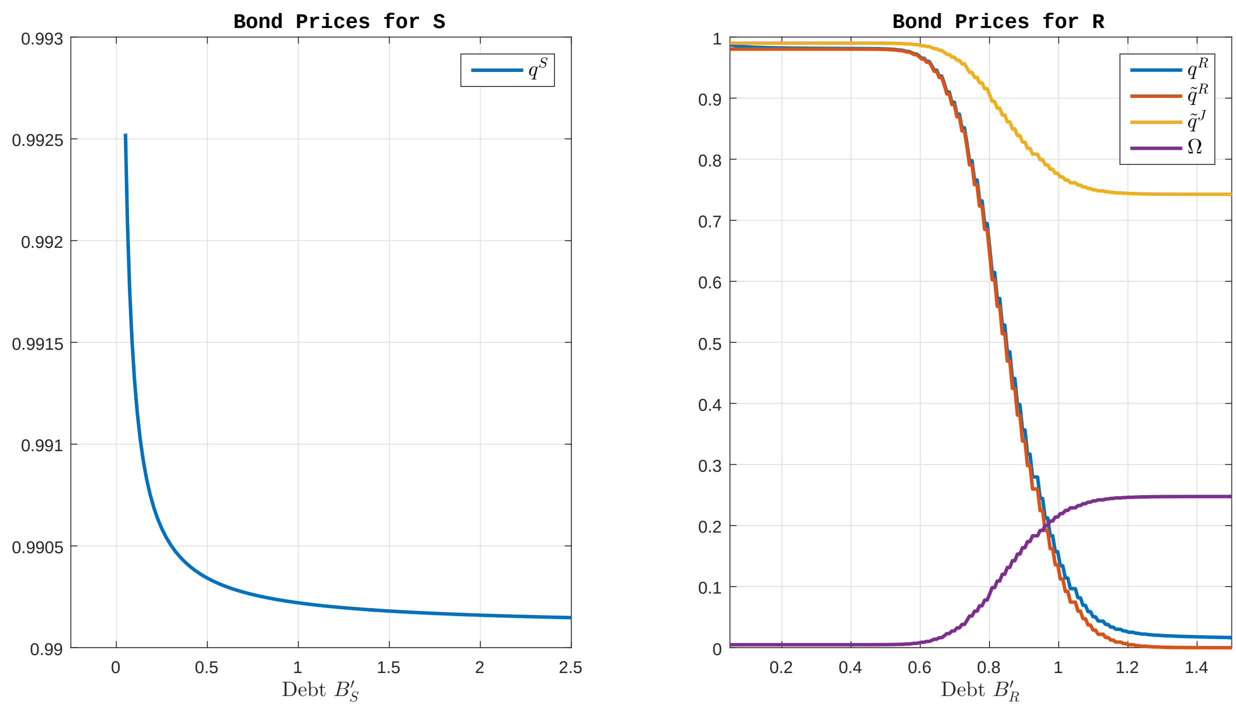

Figure 1: Bond pricing functions

$q^P_{St} > \tilde{q}_{St}$. The price of the bonds individually issued by the risky country goes down with the total amount issued. This occurs because each individually issued bond faces has more credit risk, The frictional wedge moves proportionally with the price of the risky asset. The liability wedge increases with the probability of failure of the risky bonds. Overall, the pricing wedge is increasing in b'_R because the increase in the liability wedge overcomes the decrease in the frictional wedge.

Finally, note that we can write the price differentials between the joint bond and the individually issued bonds as

$$\tilde{q}^J\left(b'_R, y_R\right) - \tilde{q}_S = -\theta_R\left(1-\lambda\right)\frac{\mathbb{E}_{y'_R|y_R}\left[d\left(b'_R, y'_R; \bar{b}\right)\right]}{1+r_J} \leq 0,$$
$$\tilde{q}^J\left(b'_R, y_R\right) - \tilde{q}_R\left(b'_R, y_R\right) = \theta_S\left(\tilde{q}_S - \tilde{q}_R\left(b'_R, y_R\right)\right) + \Omega\left(b'_R, y_R\right) \geq 0,$$

which shows that the price of the joint bond is always lower (higher) than the price of the bonds individually issued by the safe (risky) country.

Properties of bond pricing functions In addition to the changes in the borrowing behavior of countries due to the introduction of the joint bond, the changes in prices caused by changes in the joint borrowing scheme are important inputs for the welfare analysis. We now characterize several relevant comparative statics.

First, we can show that the value of $\frac{\partial q_i}{\partial \bar{b}} b_i$ is given by

$$\frac{\partial q_i}{\partial \bar{b}} b_i = \frac{\partial \tilde{q}_i}{\partial \bar{b}} b_i + \theta_i \left(\kappa \left(\tilde{q}^J - \tilde{q}_i \right) + (1-\kappa)\, \Omega \right) + \theta_i \bar{b} \left(\kappa \left(\frac{\partial \tilde{q}^J}{\partial \bar{b}} - \frac{\partial \tilde{q}_i}{\partial \bar{b}} \right) + (1-\kappa) \frac{\partial \Omega}{\partial \bar{b}} \right). \tag{14}$$

This expression captures the change in total revenue raised by a unit increase in the total issuance of the joint bond, holding constant the level of individual issuance. It is easy to show that $\frac{\partial q_R}{\partial \bar{b}} b_R$ is always positive for the risky country and that $\frac{\partial q^R}{\partial \bar{b}} b_S$ can take on any sign for the safe country. The sum across countries of $\frac{\partial q_i}{\partial \bar{b}} b_i$ has an intuitive interpretation and plays a relevant role in the welfare analysis. Because of its importance, we express the following result as a lemma.

Lemma 1. (Aggregate revenue impact of marginal joint bond issuance) *Holding constant the borrowing decisions of both countries, an increase in the level of joint bond issuance causes a change in the total amount raised of*

$$\sum_i \frac{\partial q_i}{\partial \bar{b}} b_i = \sum_i \frac{\partial \tilde{q}_i}{\partial \bar{b}} b_i + \Omega + \frac{\partial \Omega}{\partial \bar{b}} \bar{b}. \tag{15}$$

The induced change in the total amount issued has three components which do not cancel out in the aggregate. First, an increase in $\bar{b}$ changes the price of the individually issued bonds by changing the default probability. Second, increasing the level of joint bond issuance $\bar{b}$ earns the pricing wedge. Third, an increase in $\bar{b}$ will impact at the margin the pricing wedge earned by all the inframarginal units of the joint bond. Note the last two terms can be written as the pricing wedge corrected by a price impact elasticity as $\Omega \left(1 + \frac{\partial \Omega / \Omega}{\partial \bar{b} / \bar{b}} \right)$. Note also that while κ plays an important role in Equation (14), it fully cancels out in Equation (15).

We also show in the appendix that the direct effect of an increasing in borrowing by country R on the unit price per unit of bond issued by country S is given by

$$\frac{\partial q_S}{\partial b'_R} = \phi_{St} \left(\kappa \frac{\partial \tilde{q}^J}{\partial b'_R} + (1-\kappa) \frac{\partial \Omega}{\partial b'_R} \right).$$

Note that $\frac{\partial q^J}{\partial b'_R} = 0$ when $\bar{b} = 0$. This fact is crucial to show in Proposition 1 that the free-riding effects that arise through pooled issuance are second order in the vicinity of $\bar{b} = 0$. Finally, we show in the appendix that $\frac{\partial \tilde{q}_R}{\partial \bar{b}} \geq 0$. Because country S never defaults, it is trivial to show that $\frac{\partial \tilde{q}_R}{\partial \bar{b}} = 0$.

Optimality conditions It is useful to understand the determination of the optimal policies chosen by both countries. First, we focus on a period in which it is optimal for the risky country not to default. In that case, assuming that the bond pricing functions and the value function are differentiable, the following two Euler equations apply to the debt choices of the risky and the safe countries, respectively

$$u_R'(c_R)\left(q_R+\frac{\partial q_R}{\partial b_R'}b_R'\right)=\beta_R\mathbb{E}^{\mathcal{N}}_{y_R'|y_R}\left[u_R'\left(c_R'\left(b_R',y_R'\right)\right)\right], \tag{16}$$

$$u_S'(c_S)\,q_S=\beta_S\mathbb{E}_{y'|y}\left[u_S'\left(c_S'\left(b',y'\right)\right)\right]. \tag{17}$$

Equation (16) highlights that the risky country internalizes the effect of their borrowing decision on the interest rate charged by international investors. Equation (17) corresponds to the standard Euler equation of a price taking agent. Because both countries are net lenders every period, it is easy to show that high interest rates are associated with low borrowing and vice versa.

The default decision of the risky country is given by a boundary $\hat{y}\left(b_R;\overline{b}\right)$ in the space (y_R, b_R), defined by

$$v_R^c\left(b_R,\hat{y}\left(b_R;\overline{b}\right);\overline{b}\right)=v_R^d\left(\hat{y}\left(b_R;\overline{b}\right);\overline{b}\right).$$

When $y_R<\hat{y}(\cdot)$, it is optimal for the risky country to default, so $d_R=1$. When $y_R\geq\hat{y}(\cdot)$, it is optimal for the risky country not to default, so $d_R=0$.

3 Normative results

We now study how varying the size of the joint borrowing scheme $\overline{b}$ affects the welfare of both countries. We first characterize the welfare effect of a marginal change in the size the joint borrowing scheme separately for each country. We then study the aggregate welfare effects using a Kaldor-Hicks approach.[14]

Because international investors are perfectly competitive and make zero profit, they drop out of the social welfare calculation. Therefore, social welfare for arbitrary Pareto weights ζ^i in a continuation state, denoted by $W\left(\overline{b}\right)$, is given by

$$W\left(\overline{b}\right)=\sum_i\zeta^i v_i^c\left(b,y;\overline{b}\right).$$

We first study the desirability of introducing a joint borrowing scheme by characterizing the first-order effects associated with a change in the level of the joint bond around $\overline{b}=0$.[15] This characterization is relevant because it provides a simple test to qualitatively determine whether a joint borrowing scheme is welfare improving. When $\left.\frac{dW}{d\overline{b}}\right|_{\overline{b}=0}>0$, the welfare maximizing level of $\overline{b}$ is positive.[16]

[14]Our Kaldor-Hicks welfare criterion maximizes the sum of certainty equivalents. This approach is equivalent to aggregating indirect utilities using a social welfare function with arbitrary Pareto weights but allowing for ex-ante transfers between countries.

[15]We proceed under the assumption that value functions are differentiable in $\overline{b}$. See Clausen and Strub (2014) for how to provide conditions on differentiabilty in a very related environment.

[16]Similar insights can be be drawn by presenting the results in recursive form, once one notes that the forcing

We can write the change in welfare for each country, valued at an initial date 0, induced by a marginal change in the size of the joint bond $\overline{b}$, for a small amount of joint bond, as follows

$$\left.\frac{\frac{dv_R^c}{d\overline{b}}}{u_S'(c_{0R})}\right|_{\overline{b}=0} = \sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\Pi_{Rt}\frac{\partial q_{Rt}}{\partial \overline{b}} b_{Rt}\right], \tag{18}$$

$$\left.\frac{\frac{dv_S^c}{d\overline{b}}}{u_S'(c_{0S})}\right|_{\overline{b}=0} = \sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\Pi_{St}\frac{\partial q_{St}}{\partial \overline{b}} b_{St}\right] - \lambda\theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}}\left[\Pi_{St}\right], \tag{19}$$

where $\Pi_{it} \equiv \beta_i^t \frac{u_i'(c_{it})}{u_i'(c_{i0})}$ denotes country i stochastic discount factor and $\mathbb{E}_0^{\mathcal{N}}[\cdot]$ and $\mathbb{E}_0^{\mathcal{D}}[\cdot]$ are date 0 expectations over no-default and default states, respectively. We denote by $\overline{b}^*$ the size of the optimal joint borrowing scheme under Kaldor-Hicks aggregation.

Proposition 1. (A test for positive joint bond issuance) *The desirability of introducing a joint bond scheme can be determined by measuring the net present value of the change in bond revenue raised net of joint liability payments, holding borrowing policies constant. Formally, denoting by $\overline{b}^*$ the optimal size of the joint bond issuance scheme, and by $W_{\overline{b}} = \left.\frac{dW}{d\overline{b}}\right|_{\overline{b}=0}$ the marginal welfare change, as in*

$$W_{\overline{b}} \equiv \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\sum_i \Pi_{it}\frac{\partial q_{it}}{\partial \overline{b}} b_{it}\right]}_{\text{Bond Revenue Change}} > \underbrace{\lambda\theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}}\left[\Pi_{St}\right]}_{\text{Joint Liability Payment}}, \tag{20}$$

if $W_{\overline{b}} > (\leq)\, 0$, then the optimal scheme $\overline{b}^*$ is positive (negative).

Proposition 1 states that the convenience of issuing a positive amount of joint bond can be determined by comparing the increase in the net present value of the change in bond revenue, holding constant the issuance policies, after subtracting the net present value of the change in the joint liability commitments. This result highlights the importance of looking at the behavior of bond prices/yields to determine whether a joint borrowing arrangement is desirable. In particular, for a joint borrowing arrangement without joint liability, in which $\lambda = 0$, only information about bond price behavior and SDF's in non-default states is needed to assess the welfare consequences of policy changes. This is an interesting result that is not a direct application of Modigliani-Miller logic, because we are in an environment with incomplete markets and costly default. This result crucially relies instead on investors pricing bonds optimally.[17]

This characterization should hold in more general models, because it is a direct consequence of the optimizing behavior of both countries. This result provides a simple test

variables of the solution to the functional equations that $\frac{dv_R^c}{d\overline{b}}$ and $\frac{dv_S^c}{d\overline{b}}$ must satisfy are those that appear in Equations (18) and (19) – see Equations (29) and (30) in the appendix.

[17]See Alvarez and Jermann (2004) and Davila (2015) for alternative environments in which the price of specific claims are sufficient to find normative results.

to gauge policy discussions, since all the variables that determine matter to determine $\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0}$ are potentially measurable. Although Proposition 1 identifies the key sufficient statistic to determine the desirability of positive joint bond issuance, exploiting the specific structure of the model it is possible to trace back the sources of welfare gains/losses to more primitive distortions.

The expression for $\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0}$ can be further decomposed as follows

$$\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0} = \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{C}ov_i \left[\Pi_{it}, \frac{\partial q_{it}}{\partial \bar{b}} b_{it} \right] \right]}_{\text{Risk Sharing}} + \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \Omega_t^L \right] - \lambda \theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}} \left[\Pi_{St} \right]}_{\text{Joint Liability}}$$
$$+ \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \mathbb{E}_i \left[\frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} \right] \right]}_{\text{Default Change}} + \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \Omega_t^F \right]}_{\text{Frictional}}, \tag{21}$$

where Ω_t^L and Ω_t^F are defined in Equation (4). The risk sharing and joint liability components can take on any sign. The default change and the frictional components are positive.

Proposition 1 further decomposes the different effects that determine $\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0}$ into more primitive components. As opposed to the result of Proposition 1a, this decomposition relies on the underlying assumptions of the model. We show that four distinct components affect the change in welfare. We refer to the first term in Equation (21) as the *risk sharing* component. It is formally given by

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{C}ov_i \left[\Pi_t^i, \frac{\partial q_{it}}{\partial \bar{b}} b_{it} \right] \right]$$

It corresponds to the expected sum of the cross sectional covariances of individual stochastic discount factors with the pricing impact of the increase on $\bar{b}$ on the amount of debt total debt outstanding by country i. It captures the fact that the joint bond may help to channel resources through cheaper issuance towards countries in poor conditions, and vice versa. This term is positive when the direct change in prices, holding constant borrowing decisions, induced by the change in $\bar{b}$ favors the country with relatively higher marginal utility in a given date/state. This term would appear as longs as both countries are not perfectly insured and the terms of their pricing change with the $\bar{b}$. In general, the sign of this component is indeterminate.

We refer to the second term in Equation (21) as the *default change* component. It is formally given by

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \mathbb{E}_i \left[\frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} \right] \right]$$

It corresponds to the net present value of the direct change in the price of individual bonds (without accounting for behavioral responses) induced by the change in $\bar{b}$. Because in our model, $\frac{\partial \tilde{q}_{St}}{\partial \bar{b}} = 0$, only the reduction on the default probability of the risky country determines the term $\mathbb{E}_i\left[\frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it}\right]$. This term is capturing the direct gain from reducing the probability of default by the change in the scale of the joint borrowing scheme. This term would be zero in a model without default risk. This term is in general positive, since the joint bond improves the conditions of the risky country when it does not default, holding the borrowing policy constant.

We refer to the third term in Equation (21) as the *joint liability* component. It is formally given by

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\mathbb{E}_i\left[\Pi_{it}\right] \Omega_t^L\right] - \lambda\theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}}\left[\Pi_{St}\right]$$

It accounts for the net present value, measured using the relevant stochastic discount factor for each country of the joint liability commitment generated by issuing the joint bond. Given that $\Omega_t^L = \lambda\theta_R\left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right)$, it is easy to see that this component is zero when there is no joint liability. The first term measures the increase in revenue ex-ante while the second one corresponds to the payments to lenders in case of default. The welfare gain generated by the joint bond is positive when the value of the former is greater than the latter, and vice versa. In general, the sign of this component is indeterminate.

We refer to the fourth term in Equation (21) as the *frictional* component. It is formally given by

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\mathbb{E}_i\left[\Pi_{it}\right] \Omega_t^F\right]$$

It captures the gains obtained by earning the non-pecuniary benefits associated with joint issuance. Given that $\Omega_t^F = \theta_R \frac{r_R - r_J}{1+r_J}\tilde{q}_{Rt}$, it is clear that this term is positive as long as the joint bond earns a non-pecuniary benefit over the bonds issued by the risky country.

Although Proposition 1 is sufficient to determine the desirability of issuing a joint bond, it does not provide quantitative or qualitative guidance on the size of the optimal scheme. By characterizing the value of $\frac{dW}{d\bar{b}}$ for any level of $\bar{b}$ we can locally determine for each level of $\bar{b}$ whether a local change is desirably or not. Whenever $\frac{dW}{d\bar{b}} > 0$ is positive, it is optimal to locally increase the level of joint bond issuance and vice versa.

Proposition 2. (First-order welfare effects of marginal joint bond issuance) *The welfare change induced by a marginal change in the scale of the joint borrowing scheme $\bar{b}$ is given by*

$$\frac{dW}{d\bar{b}} = \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\sum_i \Pi_{it}\frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right]}_{\textit{Bond Revenue Change}} \underbrace{- \lambda\theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}}\left[\Pi_{St}\right]}_{\textit{Joint Liability Payment}}$$
$$+ \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}}\left[\Pi_{St}\frac{\partial q_{St}}{\partial b_{Rt}} b_{St}\frac{db_{Rt}}{d\bar{b}}\right]}_{\textit{Free Riding}} + \underbrace{\sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N},y_S}\left[\Delta_{St}\frac{dF\left(\hat{y}\left(b_R';\bar{b}\right)|y_S',y\right)}{d\bar{b}}\right]}_{\textit{Default Spillover}} \qquad (22)$$

where $\Pi_{it} \equiv \beta_i^t \frac{u_i'(c_{it})}{u_i'(c_{i0})}$ *denotes country i's stochastic discount factor,* $\Delta_{St} \equiv \beta_i^t \frac{v_{St}^{\mathcal{D}} - v_{St}^{\mathcal{N}}}{u_i'(c_{S0})}$ *measures the utility gap between default and no default states, while* $\mathbb{E}_{\mathcal{N}}[\cdot]$ *and* $\mathbb{E}_{\mathcal{D}}[\cdot]$ *denote expectations over no default states and default states, respectively.*

Proposition 2 provides a more general characterization of the first-order effects that emerge when varying $\bar{b}$ far from the $\bar{b} = 0$ limit. Two new terms arise relative to proposition 1. We refer to the first new term in Equation (22) as the *free riding* component. It is formally given by

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{St} \frac{\partial q_{St}}{\partial b_{Rt}} b_{St} \frac{db_{Rt}}{d\bar{b}} \right]$$

The risky country free rides on the safe country by not internalizing how its borrowing decisions affect the amount raised by the safe country through the joint. In practice, the risky country tends to overborrow, since it does not internalize the increased borrowing rates faced by the safe country. Because we have shown that $\frac{\partial q_t^S}{\partial B_t^R} < 0$ and we observe that $\frac{db_t^R}{d\bar{b}} > 0$, this term will be negative. Appropriate corrective policies, as those described in Section 5 can ameliorate the free riding effects.

We refer to the second new term in Equation (22) as the *default spillover* component. It is formally given by

$$\sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N}, y_S} \left[\Delta_{St} \frac{dF\left(\hat{y}\left(b_R'; \bar{b}\right) \middle| y_S', y\right)}{d\bar{b}} \right]$$

This term arises because the risky country does not take into account the situation of the safe country at the time of defaulting. A small change in the amount of joint bond issued directly changes the default region, which has a first effect on welfare when $\bar{b} > 0$, modulated by the difference of the safe country utility between the default and no default states Δ_{St}. With joint liability, we expect the term Δ_t^S to be negative, since country S will be in general worse off ex-post in default states having to pay for the remaining debts of country R. Without joint liability, Δ_{St} can take positive or negative values, since it may be that country S is better off by exiting the joint borrowing scheme.

The free riding and default spillover effects are small when the amount pooled is small, vanishing for low levels of joint bond issuance, because $\Delta_{St} = \frac{\partial q_t^S}{\partial B_t^R} = 0$ when $\bar{b} \to 0$. In that regard, the form of pooling analyzed in this paper is comparable to taxation, which also creates second-order welfare losses locally.[18]

We can further decompose the first-order welfare effect in Proposition 2 in six components

[18] There is scope to understand whether a supranational planner who seeks to redistribute resources and has two instruments available, distortionary taxation and bond pooling, would prefer to use a single instrument or a mix of both.

as follows

$$\frac{dW}{d\bar{b}} = \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{C}ov_i \left[\Pi_{it}, \frac{\partial q_{it}}{\partial \bar{b}} b_{it} \right] \right]}_{\text{Risk Sharing}} + \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \left(\Omega_t^L + \frac{\partial \Omega_t^L}{\partial \bar{b}} \bar{b} \right) \right] - \lambda \theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}} \left[\Pi_{St} \right]}_{\text{Joint Liability}}$$

$$+ \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \sum_i \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} \right]}_{\text{Default Change}} + \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\mathbb{E}_i \left[\Pi_{it} \right] \left(\Omega_t^F + \frac{\partial \Omega_t^F}{\partial \bar{b}} \bar{b} \right) \right]}_{\text{Frictional}} \tag{23}$$

$$+ \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{St} \frac{\partial q_{St}}{\partial b_{Rt}} b_{St} \frac{d b_{Rt}}{d \bar{b}} \right]}_{\text{Free Riding}} + \underbrace{\sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N}, y_S} \left[\Delta_{St} \frac{dF \left(\hat{y} \left(b_R'; \bar{b} \right) \middle| y_S', y \right)}{d\bar{b}} \right]}_{\text{Default Spillover}}.$$

The risk sharing and joint liability components can take on any sign. The default change and the frictional components are in general positive. The free riding component is in general negative, while the free riding default and default spillover components are also negative under natural assumptions. We would like to highlight two results

Remark 1. *(Irrelevance of free-riding and spillover effects to determine the sign of $\bar{b}$)* Free-riding effects are second order around zero net issuance. Hence, even though

Remark 2. *(Robustness of sufficient statistics)* The set of variables that determine the welfare effects of changing the scale of the joint bond issuance scheme are are invariant to a number of modifications of the environment. In particular, understanding the change in bond revenue holding constant borrowing policies will still encapsulate the relevant hold under more general preference specifications, for instance endogenous output, non-separable utility, or more general investors' pricing kernels. Because of market incompleteness, all pecuniary effects, including terms-of-trade effects as well as changes in market prices will have a first-order effect on welfare. Labor wedges, or other forms of direct externalities, can potentially add new terms to welfare assessments.

Pareto Improvements Although Propositions 1 and 2 provide sharp results on the aggregate, policies that involve Pareto improvements make a stronger case for intervention. Under which conditions can both countries be better off by entering a joint issuance scheme? We proceed setting $\lambda = 0$, since that may hurt the safe country. In that case, we can write the welfare change for each country as

$$\left.\frac{\frac{dv_R^c}{d\bar{b}}}{u_S'(c_{0R})}\right|_{\bar{b}=0} = \sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{Rt} \left(\frac{\partial \tilde{q}_R}{\partial \bar{b}} b_R + \theta_R \left(\kappa \left(\tilde{q}^J - \tilde{q}_R\right) + (1-\kappa)\,\Omega\right)\right)\right], \tag{24}$$

$$\left.\frac{\frac{dv_S^c}{d\bar{b}}}{u_S'(c_{0S})}\right|_{\bar{b}=0} = \theta_S \sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{St} \left(\kappa \left(\tilde{q}^J - \tilde{q}_S\right) + (1-\kappa)\,\Omega\right)\right], \tag{25}$$

Because we know, as shown above, that $\tilde{q}^J - \tilde{q}_R > 0$, it is clear that the risky country benefits from joint bond issuance. Because we have set $\lambda = 0$, which implies that $\Omega = \Omega^F$, we can show that for low levels of κ up to a threshold, both countries profit from issuing a joint bond.

Proposition 3. (Pareto improvement) *When there is no joint liability ($\lambda = 0$), as long as $r^J \neq r^S$, there exists a threshold $\hat{\kappa}$ for κ, such that if $\kappa < \hat{\kappa}$, both countries benefit from joint bond issuance.*

Intuitively, the bond issuance parameter κ is generating redistribution from the safe to the risky country. Low enough levels of κ guarantee that the implicit transfer to the risky country is small enough. When κ is high, the safe country does not find any gain from the joint bond scheme.

4 Quantitative results

In this section, we solve the model numerically and explore the effects of different joint bond schemes for equilibrium variables and welfare, using as guidance the theoretical results derived in the previous section.

Table 2: Baseline Parametrization

	Parameters	Values
r_R, r_S	Risk-free rates	$1\%, 2\%$
β_R, β_S	Discount factor	$0.875, 0.971$
γ_R, γ_S	Risk aversion coefficient	2
ρ_R, ρ_S	Persistence in output	$0.945, 0.945$
σ_R, σ_S	Standard deviation of output	$0.025, 0.025$
ρ_{RS}	Correlation of outputs	0
a_0, a_1, a_2	Output cost of defaulting	$-0.88, 1, 0$
θ	Probability of regaining access to markets	0.0385
	Joint Bond Parameters	
$\bar{b}$	Size of Joint Bond	$\{0, 0.1, 0.2\}$
θ_R, θ_S	Country share	$0.5, 0.5$
κ	Degree of pooled issuance	$\{0, 0.5, 1\}$
λ	Degree of joint liability	$\{0, 0.5, 1\}$

4.1 Calibration and functional forms

We adopt a quarterly calibration. Our parameter choices are summarized in Table 2. As in much of the related literature, we adopt an isoelastic period utility specification for both countries, given by

$$u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i}, \quad \text{with} \quad \gamma_i \neq 1.$$

We set $\gamma_i = 2$ for both countries, a common value in quantitative studies. Similarly, we set the international risk-free rate to $r_S = 1\%$. We assume a liquidity non-pecuniary premium earned by the safe country and the joint bond also of 1%, so $r_R = 2\%$. Given that this is a quarterly calibration, this is non-negligible frictional/safety premium.

We use data for an average small open economy as a reference for choosing the parameters that govern the endowment processes. We assume that the endowment process for the linearly detrended GDP follows

$$\log(y_t) = A\log(y_{t-1}) + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0,\Omega), \ A = \begin{pmatrix} \rho_R & \rho_{RS} \\ \rho_{RS} & \rho_S \end{pmatrix}, \text{ and } \Omega = \begin{pmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_S^2 \end{pmatrix}.$$

We assume autocorrelation coefficients of $\rho_R = \rho_S = 0.945$ and a standard deviation of innovations to output of $\sigma_R = \sigma_S = 0.025$. We assume that endowments shocks are uncorrelated, although we also explore the possibility of having correlated shocks. As in previous work, we set the probability of regaining access to bond markets θ to 0.0385, which is consistent with an average exclusion time of 6 years – see Chatterjee and Eyigungor (2012).

As in Uribe and Schmitt-Grohé (2016), we assume a flexible loss function

$$L(y) = \max\left\{0, a_0 + a_1 y + a_2 y^2\right\}, \tag{26}$$

where $a_0, a_1 \in \mathbb{R}$ and $a_2 \geq 0$, which combines the formulations of Arellano (2008) and Chatterjee and Eyigungor (2012). For our benchmark calibration, we choose default parameters and β_R to target a number of statistics displayed in Table 3. Specifically, we choose values $\beta_R = 0.875$, $a_0 = -0.88$, $a_1 = 1$ and $a_2 = 0$ – we obtain similar results by re-calibrating the model with quadratic output costs.

We choose the level of β_S to target an average ratio for $\frac{d_b}{y_b}$ of 60%. The borrowing behavior of the safe country is very sensitive to the parameter choice for β_S, as well as that one for r_S.

4.2 Joint bond effects

Tables 3 and 4 report the key statistics of the model for the the risky and safe country, respectively, for two different values of the joint bond: the status quo economy, with $\overline{b} = 0$, and an economy with $\overline{b} = 0.15$, and for different joint issuance and joint liability parameters.

The effects of changing the size of the joint bond are mostly reflected on the behavior of

Table 3: Statistics country R

	$\overline{b}$	λ	κ	Def. Frequency	$\mathbb{E}\left[\frac{b_i}{y_i}\right]$	$\mathbb{E}\left[\hat{r}_i - r_i\right]$	$\mathbb{SD}\left[\hat{r}_i - r_i\right]$	$\mathbb{C}orr[\hat{r}_i - r_i, y_i]$
Data	n/a	n/a	n/a	2.6%	58%	7.4%	2.9%	-0.64
Country R	0	n/a	n/a	2.9%	50.7%	4.7%	5.8	-0.73
	0.15	0	0	4%	57.0%	3.1%	3.3%	-0.75
		0.5	0.5	4.1%	58.2%	3.1%	3.4%	-0.74
		1	1	4.3%	59.4%	2.8%	3.1%	-0.74

Note: we denote by $\hat{r}_i$ the equilibrium rate, calculated as $\frac{1}{q_i}$. All statistics are calculated using samples of 10,000 periods.

Table 4: Statistics country S

	$\overline{b}$	λ	κ	Def. Frequency	$\mathbb{E}\left[\frac{b_i}{y_i}\right]$	$\mathbb{E}\left[\hat{r}_i - r_i\right]$	$\mathbb{SD}\left[\hat{r}_i - r_i\right]$	$\mathbb{C}orr[\hat{r}_i - r_i, y_i]$
Data	n/a	n/a	n/a	n/a	58%	7.4%	2.9%	-0.64
Country S	0	n/a	n/a	n/a	63.1%	n/a	n/a	n/a
	0.15	0	0	n/a	72.5%	-0.2%	0%	0%
		0	1	n/a	73.8	-0.1%	0%	0%
		1	1	n/a	75.2%	0%	0%	0%

Note: we denote by $\hat{r}_i$ the equilibrium rate, calculated as $\frac{1}{q_i}$. All statistics are calculated using samples of 10,000 periods.

interest rates and debt to output ratios for the risky country. The fact that country R borrows more when $\overline{b}$ increases is a form of free riding.

Figure 2 illustrates graphically the effect of varying $\overline{b}$ for different combinations of κ and λ. As expected, a high degree of joint bond issuance reduces the interest rates faced by the risky country. However, depending on the configuration of the joint bond, the interest rate faced by the safe country can increase or decrease with $\overline{b}$. In particular, when the joint issuance parameter κ is relatively large to the pricing wedge, we observe the safe country facing higher interest rates.

4.3 Normative results: comparing joint bond schemes

Figure 4 shows the change in welfare for each country for the average values of b and y. We calculate the average value of b for the economy with $\overline{b} = 0$.

We report our results in terms of consumption equivalents, which provide a cardinal welfare measure. We proceed as follows. For given states b and y, as well as for a given choice of $\overline{b}$, we define $\alpha_i\left(b, y, \overline{b}\right)$ as the constant percent increase or decrease in the lifetime consumption stream required by a country i to be as well off as if there were no joint bond. By

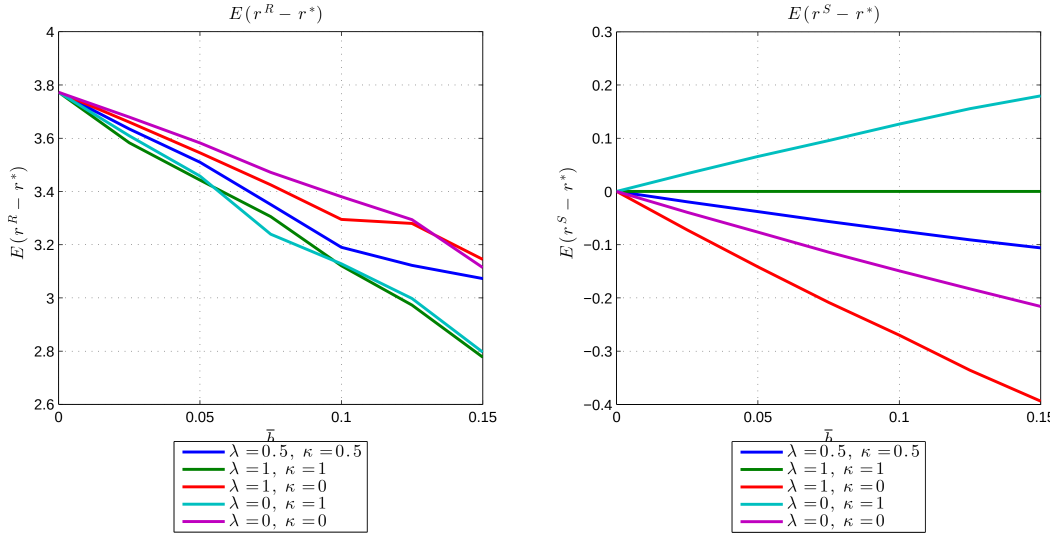

Figure 2: Change in debt-to-output ratios and interest rates

construction, this is a measure that depends on the set of state variables. Formally,[19]

$$\alpha_i\left(b,y,\bar{b}\right) = \left\{\alpha_i : v_i\left(b,y;\bar{b}\right) = (1+\alpha_i)^{1-\gamma}\, v_i\,(b,y;0)\right\} \Rightarrow \alpha_i\left(b,y,\bar{b}\right) = \left(\frac{v_i\left(b,y;\bar{b}\right)}{v_i\,(b,y;0)}\right)^{\frac{1}{1-\gamma}} - 1$$

Figure 4 provides a visual comparison of how welfare

There is an interesting interaction between the degree of joint liability and the non-pecuniary benefit. The higher the degree of joint liability the higher the price of the joint bond, which increases the value of the frictional pricing wedge, increasing the ability of both countries to do joint bond issuance at a more favorable interest rate.

[19]Note that $v_i\left(b,y;\bar{b}\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \frac{(c_{it})^{1-\gamma_i}}{1-\gamma_i}$, and that we can write

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \frac{\left((1+\alpha)\, c_{it}\right)^{1-\gamma_i}}{1-\gamma_i} = (1+\alpha)^{1-\gamma_i}\, \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \frac{(c_{it})^{1-\gamma_i}}{1-\gamma_i} = (1+\alpha)^{1-\gamma}\, v_i\left(b,y;\bar{b}\right).$$

For our particular parametrization with $\gamma = 2$, $\alpha_i\left(b,y,\bar{b}\right)$ corresponds to the proportional change in the value function of a given country:

$$\alpha_i\left(b,y,\bar{b}\right) = \frac{v_i\,(b,y;0) - v_i\left(b,y;\bar{b}\right)}{v_i\left(b,y;\bar{b}\right)}.$$

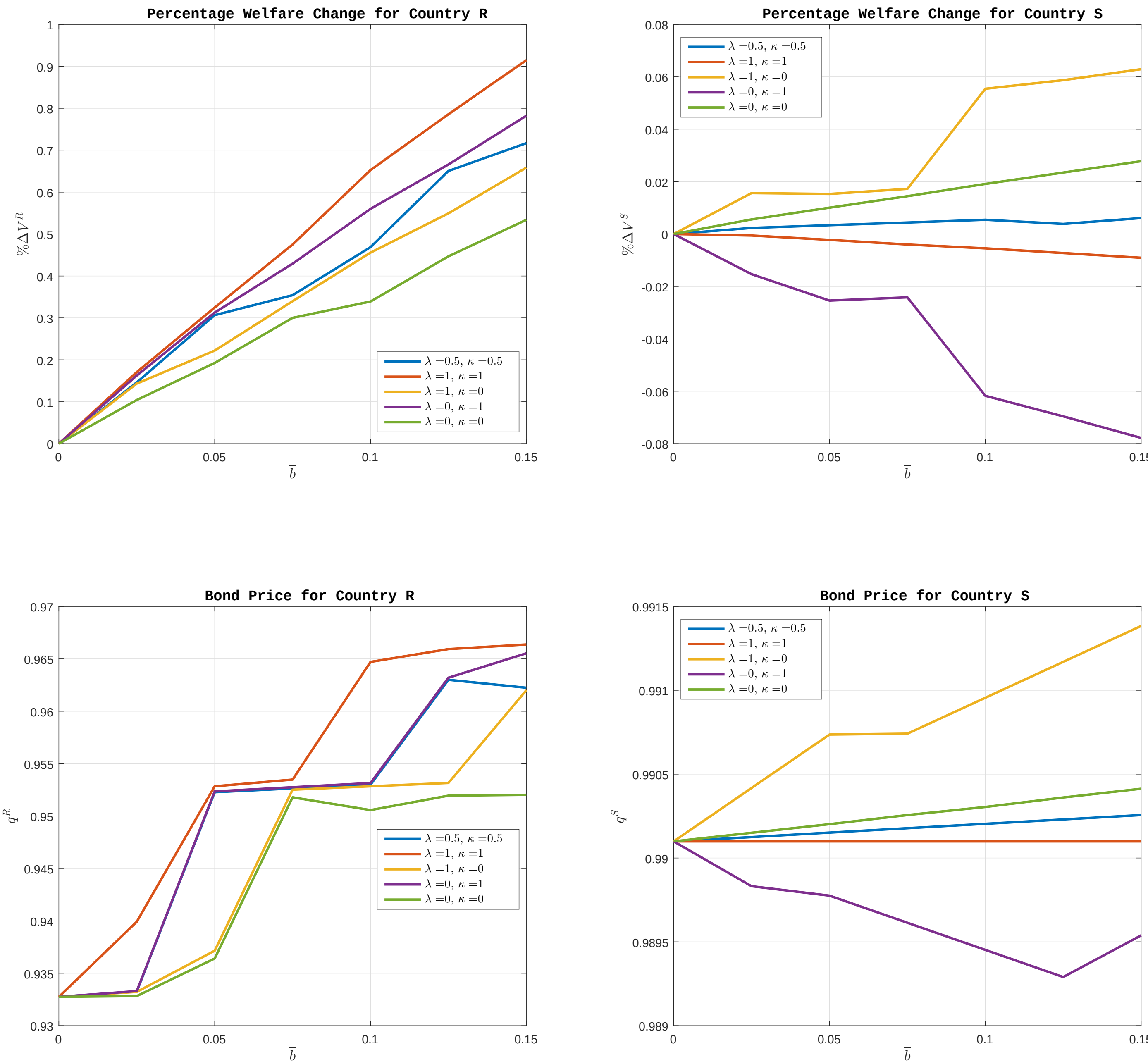

Figure 3: Effect of $\bar{b}$ on welfare and bond price functions

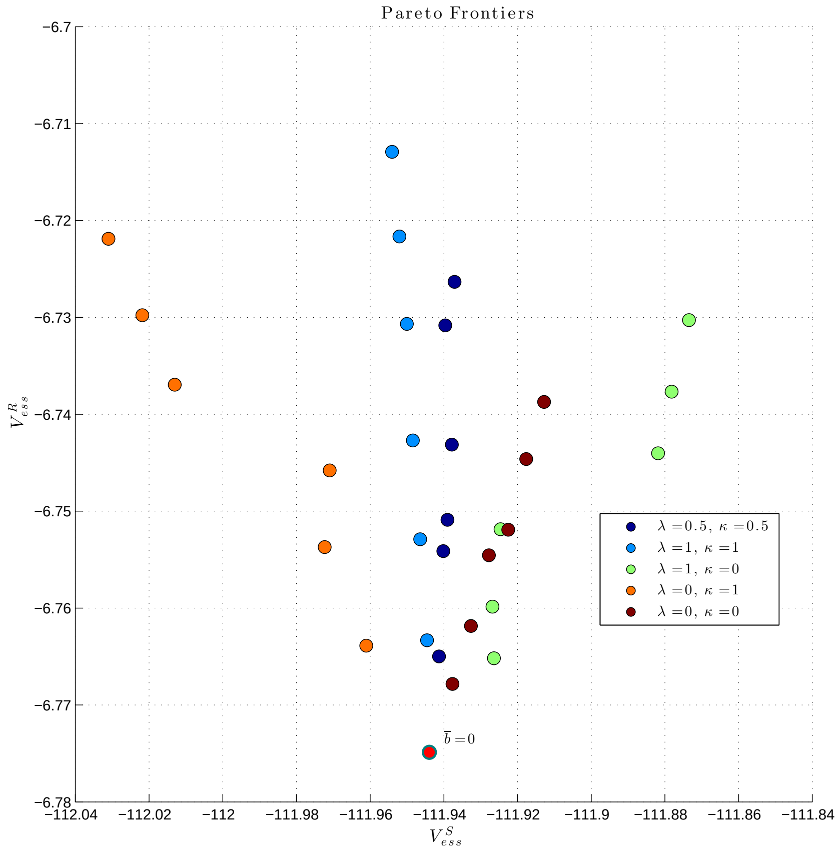

Figure 4: Welfare

5 Extensions

5.1 Direct default spillovers

In our baseline model, the default of the risky country did not have direct effects on the safe country. We now assume that the default of the risky country also generates a direct output loss for country S. Formally, this loss takes the form $\xi L(y_R)$, where $\xi \in [0,1]$ parametrizes its intensity and $L(\cdot)$ is defined in Equation (26). When $\xi = 1$, the output loss is the same for both risky and safe countries. When $\xi = 0$, the safe country does not experience any loss – this corresponds to the case studied so far. Tirole (2015) forcefully advocates for this formulation, which captures the potential cross-country negative spillovers associated with a sovereign default. In that case, we can write $v^d_S(b,y)$ as follows

$$v^d_S(b,y) = \max_{b'_S} \left\{ u_S(c_S) + \beta_S \alpha \mathbb{E}_{y'|y}\left[v_S(b',y')\right] + \beta_S(1-\alpha)\mathbb{E}_{y'|y}\left[v^d_S(b',y')\right]\right\} \tag{27}$$

$$\text{subject to} \quad c_S = y^d_S - b_S + \tilde{q}_S b'_S, \quad \text{where } y^d_S = y_S - \xi L(y_S)$$

Note that, when the risky country is excluded from financial markets, the risky country borrows at the constant risk-free rate $\tilde{q}_S$. The value of ξ modulates the strength of the direct output loss face by safe country when the risky country defaults. Equation (20) gets modified.

$$\left.\frac{\frac{dv_S^c}{d\bar{b}}}{u_S'(c_{0S})}\right|_{\bar{b}=0} = \sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{St} \frac{\partial q_{St}}{\partial \bar{b}} b_{St}\right] - \lambda \theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}} \left[\Pi_{St}\right] + \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N}, y_S} \left[\Delta_{St} \frac{dF\left(\hat{y}\left(b_R'; \bar{b}\right) | y_S', y\right)}{d\bar{b}}\right],$$

where $\Pi_{it} \equiv \beta_i^t \frac{u_i'(c_{it})}{u_i'(c_{i0})}$ denotes country i stochastic discount factor and $\mathbb{E}_0^{\mathcal{N}}[\cdot]$ and $\mathbb{E}_0^{\mathcal{D}}[\cdot]$ are expectations over no-default and default states, respectively. We denote the gap in value functions at the boundary realizations between defaulting and not by $\Delta_{St} \equiv v_S^d\left(b_t, \hat{y}_t^{dd}\right) - v_S^c\left(b_t, \hat{y}_t\right)$.

Proposition 4. (Desirability of joint borrowing scheme) *The desirability of introducing a joint borrowing scheme must include, in addition to information about the net present value of the change in bond revenue raised net of joint liability payments, the direct spillover effect caused by a change in the scale of* $\bar{b}$

$$\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0} = \underbrace{\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\sum_i \Pi_{it} \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right]}_{\text{Bond Revenue Change}} - \underbrace{\lambda \theta_R \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{D}} \left[\Pi_{St}\right]}_{\text{Joint Liability Payment}} + \underbrace{\sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N}, y_S} \left[\Delta_{St} \frac{dF\left(\hat{y}\left(b_R'; \bar{b}\right) | y_S', y\right)}{d\bar{b}}\right]}_{\text{Default Spillover}}, \tag{28}$$

If $\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0} > 0$*, it is optimal to issue a positive amount of joint bond.*

5.2 Corrective instruments

Until now, countries have been able to make borrowing choices freely. As we have characterized in Proposition 2, the change in borrowing by country R induced by increasing the scale of the joint borrowing scheme causes a welfare decreasing free-riding effect. We now describe how allowing for an additional set of corrective instruments can take care of free-riding effects.

Formally, we allow for a set of time and state contingent taxes/wedges that correct the borrowing behavior of the risky country. However, we still allow the risky country to freely decide when to default.

Proposition 5. (Optimal corrective policy) *The optimal path of corrective taxes for the risky country* τ_t^R *that internalizes the free-riding effects caused by the joint borrowing scheme is such that the following expression in Equation (22) is set to zero*

$$\sum_{t=0}^{\infty} \mathbb{E}_0^{\mathcal{N}} \left[\Pi_{St} \frac{\partial q_{St}}{\partial b_{Rt}} b_{St} \frac{db_{Rt}}{d\bar{b}}\right] + \sum_{t=1}^{\infty} \mathbb{E}_0^{\mathcal{N}, y_S} \left[\Delta_{St} \frac{dF\left(\hat{y}\left(b_R'; \bar{b}\right) \middle| y_S', y\right)}{d\bar{b}}\right]$$

Intuitively, the optimal corrective policy distorts the borrowing choices of the risky country to internalize both free riding effects. It forces the risky country to internalize the cost of more expensive borrowing to the risky country as well as the utility wedge caused in the case of default.

Importantly, when $\overline{b} = 0$, it is optimal to set $\tau_t^R = 0$: free-riding effects are zero for small interventions. More generally, there is no reason to distort the behavior of the safe country, so $\tau_t^S = 0$. The optimal corrective policy can be implemented in the form of price or quantity policies.

6 Conclusion

We have studied the positive and normative implications of joint bond issuance within a canonical model of sovereign default. Using a framework that decouples the implications of pooled issuance and joint liability, we have provided a multiple channel decomposition of the welfare effects of introducing a joint borrowing scheme.

We show that the issues of whether a joint bond scheme is desirable boils down to measuring what is the direct effect on the net present value of net revenue raised holding constant countries' borrowing choices. More generally, we have shown that a change in the level of joint bond issuance has multiple first-order effects.

Although this paper has identified several key tradeoffs associated with a joint borrowing, there is scope for much further research in this area. For instance, allowing countries to issue debt at different maturities to understand whether joint borrowing agreement are more desirable for long or short maturity bonds seems like a natural avenue for further research. Similarly, a model with richer interactions between the government and financial and real sectors could generate additional relevant insights. We leave these topics for future research. Finally, although we have framed the problem in the context of multiple sovereign countries, there is scope to apply the results in the context of different subnational authorities, for example, relating Federal and State government borrowing.

Appendix

Figure 5 shows the evolution of the sovereign yield spreads

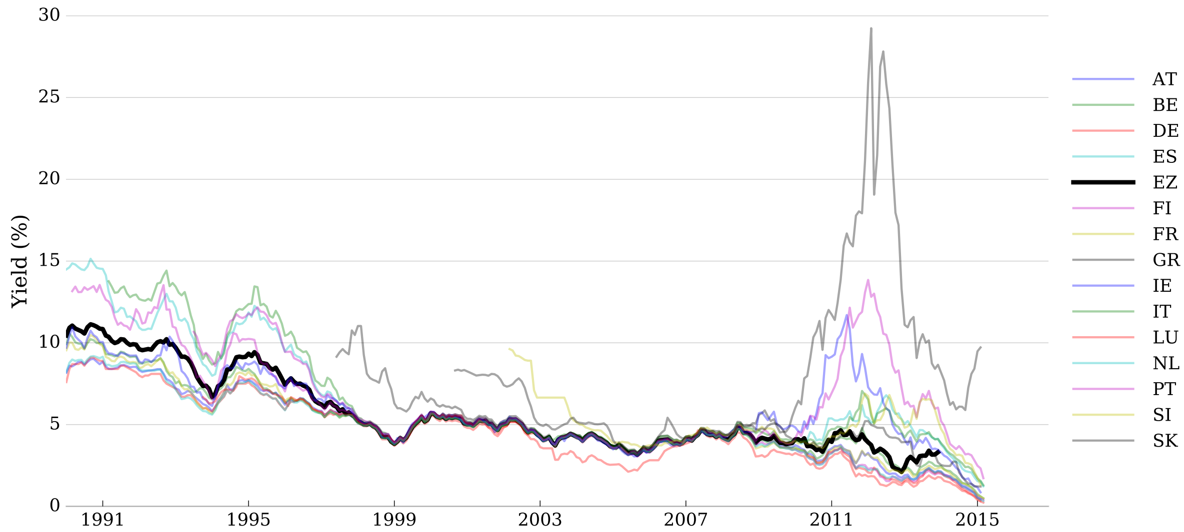

Figure 5: Sovereign bond yields (nominal yields in percentage points, 10-year bonds) for Eurozone members (1990-2014). The black line (EZ) plots the GDP-weighted average of bond yields for Eurozone members.

Proofs: Section 3

Proof of Lemma 1 (Aggregate revenue impact of marginal joint bond issuance)

We can write the partial derivative $\frac{\partial q_{it}}{\partial \bar{b}}$ as

$$\frac{\partial q_{it}}{\partial \bar{b}} = \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} + \frac{\partial \phi_{it}}{\partial \bar{b}} \left(q_{it}^{P} - \tilde{q}_{it} \right) + \phi_{it} \left(\frac{\partial q_{it}^{P}}{\partial \bar{b}} - \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} \right).$$

Using the fact that $\frac{\partial \phi_{it}}{\partial \bar{b}} b_{it} = \theta_i$, we can express $\frac{\partial q_{it}}{\partial \bar{b}} b_{it}$ as follows

$$\begin{aligned}
\frac{\partial q_{it}}{\partial \bar{b}} b_{it} &= \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} + \theta_i \left(q_{it}^{P} - \tilde{q}_{it} \right) + \phi_{it} \left(\frac{\partial q_{it}^{P}}{\partial \bar{b}} b_{it} - \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} \right) \\
&= \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} + \theta_i \left(\kappa \left(\tilde{q}_{t}^{J} - \tilde{q}_{it} \right) + (1-\kappa)\, \Omega_t \right) + \theta_i \bar{b} \left(\kappa \left(\frac{\partial \tilde{q}_{t}^{J}}{\partial \bar{b}} - \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} \right) + (1-\kappa) \frac{\partial \Omega_t}{\partial \bar{b}} \right).
\end{aligned}$$

Where we use the fact that $\phi_{it}b_{it} = \theta_i\overline{b}$, as well as the following two ways of expressing $\frac{\partial q^P_{it}}{\partial \overline{b}}$ and $\frac{\partial q^P_{it}}{\partial \overline{b}} - \frac{\partial \tilde{q}_{it}}{\partial \overline{b}}$

$$\frac{\partial q^P_{it}}{\partial \overline{b}} = \kappa\frac{\partial \tilde{q}^J_t}{\partial \overline{b}} + (1-\kappa)\frac{\partial \tilde{q}_{it}}{\partial \overline{b}} + (1-\kappa)\frac{\partial \Omega_t}{\partial \overline{b}},$$
$$\frac{\partial q^P_{it}}{\partial \overline{b}} - \frac{\partial \tilde{q}_{it}}{\partial \overline{b}} = \kappa\left(\frac{\partial \tilde{q}^J_t}{\partial \overline{b}} - \frac{\partial \tilde{q}_{it}}{\partial \overline{b}}\right) + (1-\kappa)\frac{\partial \Omega_t}{\partial \overline{b}}$$

Aggregating across countries, and combining the results just derived, we recover Equation (15) in the text

$$\begin{aligned}\sum_i \frac{\partial q_{it}}{\partial \overline{b}} b_{it} &= \sum_i \frac{\partial \tilde{q}_{it}}{\partial \overline{b}} b_{it} + \left(\kappa\left(\tilde{q}^J_t - \sum_i \theta_i \tilde{q}_{it}\right) + (1-\kappa)\,\Omega_t\right) + \overline{b}\left(\kappa\left(\frac{\partial \tilde{q}^J_t}{\partial \overline{b}} - \sum_i \theta_i \frac{\partial \tilde{q}_{it}}{\partial \overline{b}}\right) + (1-\kappa)\frac{\partial \Omega_t}{\partial \overline{b}}\right) \\ &= \sum_i \frac{\partial \tilde{q}_{it}}{\partial \overline{b}} b_{it} + \Omega_t + \overline{b}\frac{\partial \Omega_t}{\partial \overline{b}}\end{aligned}$$

Proof of Proposition 1 (A test for positive joint bond issuance)

Assuming differentiability of the value functions $v^c_R(\cdot)$ and $v^d_R(\cdot)$ in $\overline{b}$, we can express $\frac{dv_R}{d\overline{b}}$ as follows

$$\frac{dv_R}{d\overline{b}}\left(b_R, y_R; \overline{b}\right) = (1-d_R)\frac{dv^c_R\left(b_R, y_R; \overline{b}\right)}{d\overline{b}} + d_R\frac{dv^d_R\left(y_R; \overline{b}\right)}{d\overline{b}},$$

where $\frac{dv^c_R(b_R,y_R;\overline{b})}{d\overline{b}}$ and $\frac{dv^d_R(y_R;\overline{b})}{d\overline{b}}$ must satisfy, using the envelope theorem for both borrowing and defaulting decisions the following two equations:

$$\frac{dv^c_R\left(b_R, y_R; \overline{b}\right)}{d\overline{b}} = u'_R(c_R)\frac{\partial q_R\left(b'_R, y_R; \overline{b}\right)}{\partial \overline{b}} b'_R + \beta_R \mathbb{E}_{y'_R|y_R}\left[(1-d'_R)\frac{dv^c_R\left(b'_R, y'_R; \overline{b}\right)}{d\overline{b}} + d'_R\frac{dv^d_R\left(y'_R; \overline{b}\right)}{d\overline{b}}\right], \tag{29}$$

$$\frac{dv^d_R\left(y_R; \overline{b}\right)}{d\overline{b}} = \beta_R \alpha \mathbb{E}_{y'_R|y_R}\left[\frac{dv_R\left(0, y'_R; \overline{b}\right)}{d\overline{b}}\right] + \beta_R(1-\alpha)\mathbb{E}_{y'_R|y_R}\left[\frac{dv^d_R\left(y'_R; \overline{b}\right)}{d\overline{b}}\right]. \tag{30}$$

As noted in the text, Equations (29) and (30) can be interpreted as functional equations for $\frac{dv^c_R(b_R,y_R;\overline{b})}{d\overline{b}}$ and $\frac{dv^d_R(b_R,y_R;\overline{b})}{d\overline{b}}$, with a single forcing element $u'_R(c_R)\frac{\partial q_R(b'_R,y_R;\overline{b})}{\partial \overline{b}} b'_R$. Developing the solution for $\frac{dv^c_R(b_R,y_R;\overline{b})}{d\overline{b}}$ in sequence form and taking the limit when $\overline{b} \to 0$, allows us to recover Equation (18) in the text.

A similar logic applies to the safe country. We can express $\frac{dv_S}{d\overline{b}}$ as follows

$$\frac{dv_S\left(b, y; \overline{b}\right)}{d\overline{b}} = \left(1 - d_R\left(b_R, y_R; \overline{b}\right)\right)\frac{dv^c_S\left(b, y; \overline{b}\right)}{d\overline{b}} + d_R\left(b_R, y_R; \overline{b}\right)\left(\frac{dv^d_S\left(b, y; \overline{b}\right)}{d\overline{b}} - \lambda\theta^R u'_S(c_S)\right),$$

where $\frac{dv_S^c(b,y;\bar{b})}{d\bar{b}}$ and $\frac{dv_S^d(b,y;\bar{b})}{d\bar{b}}$ must satisfy,

$$\begin{aligned}\frac{dv_S^c\left(b,y;\bar{b}\right)}{d\bar{b}} &= u_S'\left(c_S\right)\frac{\partial q_S\left(b_R',y_R;\bar{b}\right)}{\partial \bar{b}}b_S' + u_S'\left(c_S\right)\frac{\partial q_S\left(b_R',y_R;\bar{b}\right)}{\partial b_R'}\frac{db_R'}{d\bar{b}}b_S' \\ &+ \beta_S \mathbb{E}_{y'|y}\left[\frac{dv_S^c\left(b',y';\bar{b}\right)}{d\bar{b}}\right] + \beta_S \mathbb{E}_{y'|y}\left[\frac{dv_S^d\left(b',y'^{,dd};\bar{b}\right)}{d\bar{b}}\right] + \beta_S\int_0^\infty \left(v_S^d\left(b',\hat{y}'^{,dd}\right) - v_S^c\left(b',\hat{y}\right)\right)\frac{d\hat{y}}{d\bar{b}}f\left(\hat{y},y_S'|y\right)dy_S', \\ \frac{dv_S^d\left(b,y;\bar{b}\right)}{d\bar{b}} &= u_S'\left(c_S\right) + \beta_S\alpha\mathbb{E}_{y'|y}\left[\frac{dv_S^c\left(b,y;\bar{b}\right)}{d\bar{b}}\right] + \beta_S\left(1-\alpha\right)\mathbb{E}_{y'|y}\left[\frac{dv_S^d\left(b',y';\bar{b}\right)}{d\bar{b}}\right],\end{aligned}$$

where $\hat{y}^{dd} = \left(y_R, y_S - \lambda\theta^R\bar{b}\right)$. We exploit the fact that $\frac{d\hat{y}}{d\bar{b}}f\left(\hat{y},y_S'|y\right) = \frac{d\hat{y}}{d\bar{b}}f\left(\hat{y},y_S'|y\right)f\left(y_S'\right) = \frac{dF\left(\hat{y}|y_S',y\right)}{d\bar{b}}f\left(y_S'\right)$, which allows us to write

$$\int_0^\infty \left(v_S^d\left(b,\hat{y}^{dd}\right) - v_S^c\left(b,\hat{y}\right)\right)\frac{d\hat{y}}{d\bar{b}}f\left(\hat{y},y_S'|y\right)dy_S' = \mathbb{E}^{\mathcal{N},y_S}\left[\left(v_S^d\left(b,\hat{y}^{dd}\right) - v_S^c\left(b,\hat{y}\right)\right)\frac{dF\left(\hat{y}|y_S',y\right)}{d\bar{b}}\right]$$

Developing the solution for $\frac{dv_S^c(b_S,y_S;\bar{b})}{d\bar{b}}$ in sequence form and taking the limit when $\bar{b} \to 0$, allows us to recover Equation (19) in the text. Note that this derivation crucially makes use of the fact that

$$\lim_{\bar{b}\to 0} v_S^d\left(b,\hat{y}^{dd};\bar{b}\right) = \lim_{\bar{b}\to 0} v_S^c\left(b,\hat{y};\bar{b}\right)$$

Adding up the marginal welfare change for both countries, normalized by the the date 0 value of marginal utility, allows us to write

$$\left.\frac{\frac{dv_R^c}{d\bar{b}}}{u_S'\left(c_{0R}\right)}\right|_{\bar{b}=0} + \left.\frac{\frac{dv_S^c}{d\bar{b}}}{u_S'\left(c_{0S}\right)}\right|_{\bar{b}=0} = \sum_{t=0}^{\infty}\mathbb{E}_0^{\mathcal{N}}\left[\sum_i \Pi_{it}\frac{\partial q_{it}}{\partial \bar{b}}b_{it}\right]\lambda\theta_R\sum_{t=1}^{\infty}\mathbb{E}_0^{\mathcal{D}}\left[\Pi_{St}\right],$$

which immediately yields the result in Equation (20).

Proof of Proposition 2 (First-order welfare effects of marginal joint bond issuance)

We can write

$$\frac{\frac{dW^S}{d\bar{b}}}{u_S'\left(c_{S0}\right)} = \sum_t\left(\mathbb{E}_{\mathcal{N}}\left[\Pi_t^S\left(\frac{\partial q_t^S}{\partial \bar{b}}B_t^S + \frac{\partial q_t^S}{\partial B_t^R}B_t^S\frac{dB_t^R}{d\bar{b}}\right)\right] - \lambda\theta_R\mathbb{E}_{\mathcal{D}}\left[\Pi_t^S\right] + \mathbb{E}_{\mathcal{D}}\left[\Delta_t^S\left(\frac{\partial \tilde{q}_{t-1}^R}{\partial \bar{b}} + \frac{\partial \tilde{q}_{t-1}^R}{\partial B_{t-1}^R}\frac{dB_{t-1}^R}{d\bar{b}}\right)\right]\right) \quad (31)$$

$$\frac{\frac{dW^R}{d\bar{b}}}{u_R'\left(c_{R0}\right)} = \sum_t \mathbb{E}_{\mathcal{N}}\left[\Pi_t^R\frac{\partial q_t^R}{\partial \bar{b}}B_t^R\right], \quad (32)$$

Simply adding up Equations (31) and (32), and using the fact that $\xi^i U'\left(C_{i0}\right)$ is constant, because of the Kaldor-Hicks criterion, yields a value of $\frac{dW}{d\bar{b}}$ given by

$$
\begin{aligned}
\frac{dW}{d\bar{b}} = & \sum_t \mathbb{E}_{\mathcal{N}}\left[\sum_i \Pi_t^i \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right] + \sum_t \mathbb{E}_{\mathcal{N}}\left[\Pi_t^S \frac{\partial q_t^S}{\partial B_t^R} B_t^S \frac{dB_t^R}{d\bar{b}}\right] - \sum_t \lambda\theta_R \mathbb{E}_{\mathcal{D}}\left[\Pi_t^S\right] \\
& + \sum_t \mathbb{E}_{\mathcal{D}}\left[\Delta_t^S\left(\frac{\partial \tilde{q}_{t-1}^R}{\partial \bar{b}} + \frac{\partial \tilde{q}_{t-1}^R}{\partial B_{t-1}^R}\frac{dB_{t-1}^R}{d\bar{b}}\right)\right] \qquad (33)
\end{aligned}
$$

2We use the notation $\mathbb{E}_i\left[\cdot\right]$ to represent a cross-sectional average $\sum_i$, to be able to write[20]

$$
\begin{aligned}
\mathbb{E}_i\left[\Pi_t^i \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right] &= \mathbb{E}_i\left[\Pi_t^i\right]\mathbb{E}_i\left[\frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right] + \mathbb{C}ov_i\left[\Pi_t^i, \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right] \\
&= \mathbb{E}_i\left[\Pi_t^i\right]\left(\sum_i \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it} + \Omega_t + \frac{\partial \Omega_t}{\partial \bar{b}}\bar{b}\right) + \mathbb{C}ov_i\left[\Pi_t^i, \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right]
\end{aligned}
$$

Were the second line follows from lemma 1. Therefore, we can write the marginal change in welfare as1a) Starting from Equation 33 we can write

$$
\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0} = \sum_t \mathbb{E}_{\mathcal{N}}\left[\sum_i \Pi_t^i \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right] - \sum_t \lambda\theta_R \mathbb{E}_{\mathcal{D}}\left[\Pi_t^S\right]
$$

Take the limit in Equation (23) when $\bar{b} \to 0$ yields the result. The $\bar{b} \to 0$ limit relies on the fact that $\left.\frac{\partial q_t^S}{\partial B_t^R}\right|_{\bar{b}=0}$, shown in Equation 34 above, and also uses the fact that, $\left.\Delta_t^S\right|_{\bar{b}=0} = 0$, which trivially follows from the definition of Δ_t^S.

We can further decompose $\sum_t \mathbb{E}_{\mathcal{N}}\left[\sum_i \Pi_t^i \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right]$ as above to find

$$
\begin{aligned}
\left.\frac{dW}{d\bar{b}}\right|_{\bar{b}=0} = & \underbrace{\sum_t \mathbb{E}_{\mathcal{N}}\left[\mathbb{C}ov_i\left[\Pi_t^i, \frac{\partial q_{it}}{\partial \bar{b}} b_{it}\right]\right]}_{\text{Risk Sharing}} + \underbrace{\sum_t \mathbb{E}_{\mathcal{N}}\left[\mathbb{E}_i\left[\Pi_t^i\right]\Omega_t^L\right] - \lambda\theta_R \sum_t \mathbb{E}_{\mathcal{D}}\left[\Pi_t^S\right]}_{\text{Joint Liability}} \\
& + \underbrace{\sum_t \mathbb{E}_{\mathcal{N}}\left[\mathbb{E}_i\left[\Pi_t^i\right]\sum_i \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} b_{it}\right]}_{\text{Default Change}} + \underbrace{\sum_t \mathbb{E}_{\mathcal{N}}\left[\mathbb{E}_i\left[\Pi_t^i\right]\Omega_t^F\right]}_{\text{Frictional}},
\end{aligned}
$$

where all the terms are assessed at $\bar{b} = 0$.

Proofs: Section 5

(to be included)

[20]We are normalizing W in this step too.

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Online Appendix

A Additional results

Pricing relations

Note that $q_{it} - \tilde{q}_{it}$ can be written as

$$q_{it} - \tilde{q}_{it} = \phi_{it}\left(q_{it}^P - \tilde{q}_{it}\right) = \phi_{it}\left(\kappa\left(\tilde{q}_t^J - \tilde{q}_{it}\right) + (1-\kappa)\,\Omega_t\right),$$

where we use the fact that $q_{it}^P - \tilde{q}_{it} = \kappa\left(\tilde{q}_t^J - \tilde{q}_{it}\right) + (1-\kappa)\,\Omega_t$.

Letting $r_R \neq r_S \neq r_J$, the pricing wedge can be derived as follows. The bond price functions for the individually issued bonds correspond to Equations (10) and (11) in the text,[21] while the price of the joint bond can be written as

$$\begin{aligned}
\tilde{q}_t^J &= \frac{1}{1+r_J}\left(\theta_S + \theta_R\left(1-(1-\lambda)\,\mathbb{E}_t\left[d_{R,t+1}\right]\right)\right) \\
&= \frac{1}{1+r_J}\left(\theta_S + \theta_R\left(\lambda + (1-\lambda)\left(1-\mathbb{E}_t\left[d_{R,t+1}\right]\right)\right)\right) \\
&= \theta_S\frac{1+r_S}{1+r_J}\tilde{q}_{St} + \theta_R\left(\lambda\frac{1}{1+r_J} + (1-\lambda)\frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right)
\end{aligned}$$

We can therefore derive the pricing wedge as

$$\begin{aligned}
\Omega_t &= \tilde{q}_t^J - \sum\theta_i\tilde{q}_{it} \\
&= \theta_S\frac{1+r_S}{1+r_J}\tilde{q}_{St} + \theta_R\left(\lambda\left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right) + \frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right) - \theta_S\tilde{q}_{St} - \theta_R\tilde{q}_{Rt} \\
&= \left(\frac{1+r_S}{1+r_J} - 1\right)\theta_S\tilde{q}_{St} + \theta_R\lambda\left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right) + \left(\frac{1+r_R}{1+r_J} - 1\right)\theta_R\tilde{q}_{Rt} \\
&= \underbrace{\theta_S\frac{r_S - r_J}{1+r_J}\tilde{q}_{St} + \theta_R\frac{r_R - r_J}{1+r_J}\tilde{q}_{Rt}}_{\Omega_t^F \text{ (Frictional Wedge)}} + \underbrace{\theta_R\lambda\left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J}\tilde{q}_{Rt}\right)}_{\Omega_t^L \text{ (Liability Wedge)}} \\
&= \theta_S\frac{r_S - r_J}{1+r_J}\tilde{q}_{St} + \theta_R\frac{r_R - r_J}{1+r_J}\tilde{q}_{Rt} + \theta_R\lambda\frac{\mathbb{E}_t\left[d_{t+1}^R\right]}{1+r_J}
\end{aligned}$$

We can therefore decompose the pricing wedge as $\Omega_t = \Omega_t^F + \Omega^L$. When there is no joint

[21] Note that we can write

$$\tilde{q}_{Rt}\left(b_R', y_R; \bar{b}\right) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(1-d_{R,t+1}\right) f\left(y_R', y_S'\right) dy_R' dy_S'}{1+r_R} = \frac{\int_{-\infty}^{\infty}\int_{\hat{y}\left(b_R';\bar{b}\right)}^{\infty} f\left(y_R', y_S'\right) dy_R' dy_S'}{1+r_R}$$

liability, $\lambda = 0$ and $\Omega_t = \theta_S \frac{r_S - r_J}{1+r_J} \tilde{q}_{St} + \theta_R \frac{r_R - r_J}{1+r_J} \tilde{q}_{Rt}$. When all bonds are priced according to the same pricing kernel, $r_J = r_S = r_R$ and $\Omega_t = \lambda \theta_R \left(\frac{1}{1+r_S} - \tilde{q}_{Rt} \right)$.

Following Assumption 1, when $r_J = r_S < r_R$, the price of the joint bond is given by

$$\tilde{q}_t^J = \theta_S \tilde{q}_{St} + \theta_R \left(\lambda \frac{1}{1+r_J} + (1-\lambda) \frac{1+r_R}{1+r_J} \tilde{q}_{Rt} \right).$$

Similarly, the pricing wedge Ω_t corresponds to

$$\Omega_t = \underbrace{\theta_R \frac{r_R - r_J}{1+r_J} \tilde{q}_{Rt}}_{\Omega_t^F \text{ (Frictional Wedge)}} + \underbrace{\lambda \theta_R \left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J} \tilde{q}_{Rt} \right)}_{\Omega_t^L \text{ (Liability Wedge)}},$$

and including the wedge pricing impact $\Omega_t + \frac{\partial \Omega_t}{\partial \bar{b}} \bar{b}$ can be written as

$$\Omega_t + \frac{\partial \Omega_t}{\partial \bar{b}} \bar{b} = \underbrace{\theta_R \frac{r_R - r_J}{1+r_J} \left(\tilde{q}_{Rt} + \frac{\partial \tilde{q}_{Rt}}{\partial \bar{b}} \bar{b} \right)}_{\Omega_t^F + \frac{\partial \Omega_t^F}{\partial \bar{b}} \bar{b}} + \underbrace{\theta_R \lambda \left(\frac{1}{1+r_J} - \frac{1+r_R}{1+r_J} \left(\tilde{q}_{Rt} + \frac{\partial \tilde{q}_{Rt}}{\partial \bar{b}} \bar{b} \right) \right)}_{\Omega_t^L + \frac{\partial \Omega_t^L}{\partial \bar{b}} \bar{b}}.$$

Note that $\frac{\partial \Omega_t}{\partial \bar{b}} < 0$, because increasing the total amount issued reduces the default probability of the risky country which reduces the rationale for joint bond issuance. We can express the change in the pricing wedge with respect to b_R as

$$\frac{\partial \Omega}{\partial b_R} = \theta_R \left(\frac{r_R - r_J}{1+r_J} - \lambda \frac{1+r_R}{1+r_J} \right) \frac{\partial \tilde{q}_R}{\partial b_R}$$

When $\lambda = 0$, $\frac{\partial \Omega}{\partial b_R}$ is positive. When $\lambda = 1$, $\frac{\partial \Omega}{\partial b_R} = -\theta_R \frac{\partial \tilde{q}_R}{\partial b_R}$.

There are two other important sensitivities. First, we find the sensitivity of country i bond prices to the borrowing choices of country $-i$, which is given by:

$$\frac{\partial q_{it}}{\partial b_{-it}} = \phi_{it} \frac{\partial q_{it}^P}{\partial b_{-it}}, \quad \text{where} \quad \frac{\partial q_{it}^P}{\partial b_{-it}} = \kappa \frac{\partial \tilde{q}_t^J}{\partial b_{-it}} + (1-\kappa) \frac{\partial \Omega_t}{\partial b_{-it}}$$

Combining both equations we find that

$$\frac{\partial q_{it}}{\partial b_{-it}} = \phi_{it} \left(\kappa \frac{\partial \tilde{q}_t^J}{\partial b_{-it}} + (1-\kappa) \frac{\partial \Omega_t}{\partial b_{-it}} \right) \tag{34}$$

Note that $\frac{\partial q_{it}}{\partial b_{-it}}$ is 0 when $\bar{b} = 0$.

Second, the own borrowing choice price sensitivity is given by

$$\frac{\partial q_{it}}{\partial b_{it}} = \frac{\partial \tilde{q}_{it}}{\partial b_{it}} + \frac{\partial \phi_{it}}{\partial b_{it}} \left(q^P_{it} - \tilde{q}_{it} \right) + \phi_{it} \left(\frac{\partial q^P_{it}}{\partial b_{it}} - \frac{\partial \tilde{q}_{it}}{\partial b_{it}} \right)$$

Using the fact that $\frac{\partial \phi_{it}}{\partial b_{it}} b_{it} = -\phi_{it}$, we can write:

$$\frac{\partial q_{it}}{\partial b_{it}} b_{it} = \frac{\partial \tilde{q}_{it}}{\partial b_{it}} b_{it} - \phi_{it} \left(q^P_{it} - \tilde{q}_{it} \right) + \phi_{it} \left(\frac{\partial q^P_{it}}{\partial b_{it}} b_{it} - \frac{\partial \tilde{q}_{it}}{\partial b_{it}} b_{it} \right)$$

Which implies that

$$q_{it} + \frac{\partial q_{it}}{\partial b_{it}} b_{it} = \tilde{q}_{it} + \frac{\partial \tilde{q}_{it}}{\partial b_{it}} b_{it} + \phi_{it} \left(\frac{\partial q^P_{it}}{\partial b_{it}} b_{it} - \frac{\partial \tilde{q}_{it}}{\partial b_{it}} b_{it} \right), \tag{35}$$

where

$$\frac{\partial q^P_{it}}{\partial b_{it}} = \kappa \frac{\partial \tilde{q}^J_t}{\partial b_{it}} + (1-\kappa) \left(\frac{\partial \tilde{q}_{it}}{\partial b_{it}} + \frac{\partial \Omega_t}{\partial b_{it}} \right)$$

Equation (35) is the marginal revenue per unit of bond issued by country i. It is a key input to the decisions of country R. As expected, when $\bar{b} = 0$, the marginal revenue is identical to the individual one. Finally, using Equation (4) in the text, we can write $\tilde{q}^J_t = \sum_i \theta_i \tilde{q}_{it} + \Omega_t$, which allows us to express $\frac{\partial \tilde{q}^J_t}{\partial \bar{b}}$ as $\frac{\partial \tilde{q}^J_t}{\partial \bar{b}} = \sum_i \theta_i \frac{\partial \tilde{q}_{it}}{\partial \bar{b}} + \frac{\partial \Omega_t}{\partial \bar{b}}$

B Generalizations

B.1 More general joint borrowing schemes

In principle, we could allow for country specific pooled issuance parameters κ_i, as well as for a distribution of the pricing wedge that is country specific. In that case

$$q^P_{it} = \kappa_i \tilde{q}^J_t + (1-\kappa_i) \left(\tilde{q}_{it} + \chi_i \Omega_t \right),$$

where the relevant pricing wedge that guarantees that $\sum_i \theta_i q^P_{it} = \tilde{q}^J_t$ is defined as

$$\Omega_t = \frac{\sum_i \theta_i (1-\kappa_i) \left(\tilde{q}^J_t - \tilde{q}_{it} \right)}{1 - \sum_i \theta_i \chi_i \kappa_i}.$$

When $\kappa_i = \kappa$ and χ_i is such that $\sum_i \theta_i \chi_i = 1$ we recover the expression of the pricing wedge described in the text in Equation (4).

An alternative possibility is to distribute the frictional and liability wedges according to different shares, for instance, as in

$$q_{it}^{P} = \kappa_i \tilde{q}_t^{J} + (1-\kappa_i)\left(\tilde{q}_{it} + \chi_i^L \Omega_t^L + \chi_i^F \Omega_t^F\right)$$

B.2 Positive net foreign asset position

In practice, since for natural parametrizations countries are borrow significant amounts, the constraint $b_{it} \geq \theta_i \overline{b}$ is not binding. Conceptually, it is easy to allow for positive savings in this framework. When $b_{it} > 0$, country i is a net borrower, while when $b_{it} < 0$, country i is a net saver. When $b_{it} > \theta_i \overline{b}$, country i individually issues $b_{it} - \theta_i \overline{b}$ unit bonds. In that case, a fraction $\phi_{it} = \frac{\theta_i \overline{b}}{b_{it}}$ of borrowing is done through the joint bond, while the remaining $1 - \phi_{it}$ is issued individually. When $b_{it} \leq \theta_i \overline{b}$, country i saves the positive amount $\theta_i \overline{b} - b_{it}$ in international markets at the appropriate risk-free rate. Figure 6 illustrates the borrowing decision in this more general case.

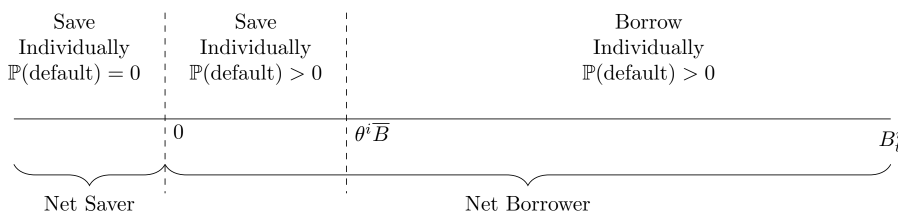

Figure 6: Illustration of borrowing/saving position

In this case, we assume that creditors can seize country R foreign assets at default. Regarding the determination of the pricing schedules, we assume that $\tilde{q}_{it}$ can be imputed even when there are no individually issued bonds outstanding. We can think that each country is issuing a small amount of those – close to zero – and that prices can be inferred from those. In this case, the equivalent to Equation (1) in the text is

$$q_{it} b_{it} = q_{it}^{P} \theta_i \overline{b} + \tilde{q}_{it} \max\left\{b_{it} - \theta_i \overline{b}, 0\right\} + \frac{1}{1+r_i} \min\left\{b_{it} - \theta_i \overline{b}, 0\right\} \tag{36}$$

Or alternatively, $q_{it} = \phi_{it} q_{it}^{P} + (1-\phi_{it})\, \hat{q}_t^i$, where $\hat{q}_t^i = \tilde{q}_{it} \mathbb{I}\left[b_{it} \geq \theta_i \overline{b}\right] + \frac{1}{1+r_i} \mathbb{I}\left[b_{it} < \theta_i \overline{b}\right]$.

C Numerical solution

We solve the model recursively using a standard discretization and value function iteration procedure. Solving the model entails finding value functions, policy functions, and bond price functions for both the risky and the safe country. We use a one-loop algorithm to solve the problem of the risky country, as described in Hatchondo, Martinez and Sapriza (2010), with at least 200 points for the endowment realization of the risky country and 200 grid points for its debt choices. We use 100 points for the endowment realization of the safe country and 200 grid points for its debt. We proceed as follows:

1. Guess an initial value function for the risky country, $v_R(b_R, y_R)$, as well as a pricing function, $\tilde{q}_R(\cdot)$.

2. At each pair (b_R, y_R), update $v_R^d(b_R, y_R)$ and $v_R^c(b_R, y_R)$.

3. Update $v_R(b_R, y_R)$, the optimal default rule, the optimal borrowing policy, and the relevant pricing functions.

4. Check for convergence. If no convergence, go back to step 2. If the model converges, stop.

5. Conditional on the optimal savings policy and pricing function for the risky country, guess an initial value function for the safe country, $v_S(b, y)$.

6. At each pair of vectors (b, y), update $v_S^d(b, y)$ and $v_S^c(b, y)$.

7. Update $v_S(b, y)$ and the optimal borrowing policy for the safe country.

8. Check for convergence. If no convergence, go back to step 6. If converged, stop.

Table 5 We use the following discretization choices. For the safe country.

Table 5: Discretization choices

Parameter		Values
$n_y^R \times n_y^S$	Grid points y_i	200×200
$n_b^R \times n_b^S$	Grid points b_i	100×100
$\left[\underline{y}, \overline{y}\right]$	Output range	[0.65]
$\left[\underline{b}, \overline{b}\right]$	Debt range	$\left[\theta_S \overline{b}, 2.5\right] \times \left[\theta_R \overline{b}, 1.5\right]$