# **Discussion** of
# Advertising Arbitrage
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- **This paper:** A theory of why arbitrageurs do advertise their arbitrage opportunities
  - Mechanism: revealing information makes arbitrage profitable through price convergence
- No previous theoretical research on this topic
  - Some relation to work on information disclosure/information acquisition (different emphasis)

# Outline of discussion

1. Describe environment
2. Revisit results
   - Highlight critical assumptions
   - Comments
3. Conclusion

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- Borrowing and short sales constraints $\Rightarrow |y_i|$ bounded **(key)**

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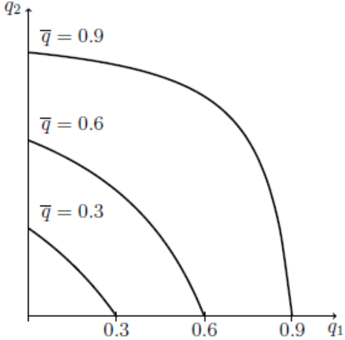
Figure 2: The arbitrageur's advertising possibility frontier.



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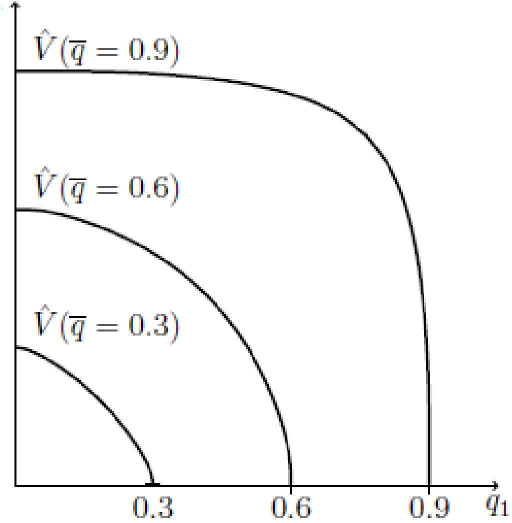
- Indifference curves $\Rightarrow$ **Main Insight**
  - *Complementarity* between portfolio choices and advertising



# Results basic model

1. Arbitrageurs concentrate advertising in a single asset
   - How robust is this result?
     - True for risk averse arbitrageurs with linear advertising frontier
     - True for CARA arbitrageurs with CARA technology
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3. More a) advertisable, b) mispriced and c) "prone to converge" assets are advertised by a risk neutral arbitrageur
   - Risk neutral limit makes results look somewhat trivial: arbitrageurs only choose to invest in one asset, hence they only advertise that asset

# More comments

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   - Fact: we only see advertising for short positions, not for long positions $\Rightarrow$ But the model is *symmetric* (counterfactual)
   - Endogenous prediction of the model if short-sale constraints are more binding than borrowing constraints **(interesting insight)**

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3. Persistent versus one-time arbitrage opportunities
   - Should affect the results

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2. Reputation
   - Partially addresses possible concerns about lying
     - Weakens results $\Rightarrow$ Advertising only occurs in some equilibria.
   - Importance of horizon of arbitrageurs for predictions

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- Some extra work could be done sharpening required assumptions
  - In *theory*. Example: necessary and sufficient conditions for non-convexities/strong complementarities
  - In *practice*. Example: which actual markets/situations verify those conditions