ECON 500a
General Equilibrium and Welfare Economics
General Static Production Economy

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Course Outline

- Block 1: Static Exchange Economies
- Block 2: Static Production Economies
- Block 3: Dynamic Stochastic Economies

Outline: Static Production Economies

1. Elementary Static Production Economies
2. General Static Production Economy
   - Physical Environment
   - Competitive Equilibrium
3. Efficiency and Welfare
4. Applications

General Static Production Economy

- This economy nests everything we have seen!
  - Consistent notation! $\rightarrow$ same as Dávila and Schaab (2024)
- $I \geq 1$ individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$
- $J \geq 1$ goods, indexed by $j, \ell \in \mathcal{J} = \{1, \ldots, J\}$
- $F \geq 1$ factors, indexed by $f \in \mathcal{F} = \{1, \ldots, F\}$

General Static Production Economy

- Physical environment:

$$V^i = u^i\left(\left\{c^{ij}\right\}_{j\in\mathcal{J}}, \left\{n^{if,s}\right\}_{f\in\mathcal{F}}\right) \qquad \text{(Preferences)}$$

$$y^{j,s} = G^j\left(\left\{x^{j\ell}\right\}_{\ell\in\mathcal{J}}, \left\{n^{jf,d}\right\}_{f\in\mathcal{F}}\right) \qquad \text{(Technologies)}$$

$$y^{j,s} + \bar{y}^{j,s} = c^j + x^j \qquad \text{(Resource constraint: goods)}$$

$$n^{f,s} + \bar{n}^{f,s} = n^{f,d} \qquad \text{(Resource constraint: factors)}$$

- $x^{j\ell}$: good ℓ used to produce good j
- $c^j = \sum_i c^{ij}$, $x^j = \sum_\ell x^{\ell j}$, $\bar{y}^{j,s} = \sum_i \bar{y}^{ij,s}$
- $n^{f,s} = \sum_i n^{if,s}$, $\bar{n}^{f,s} = \sum_i \bar{n}^{if,s}$, $n^{f,d} = \sum_j n^{jf,d}$
- An allocation is $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$
  - Dimension: $IJ + IF + J^2 + JF + J$

Competitive Equilibrium: Definition

- A *competitive equilibrium* is an allocation $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$, prices $\left\{p^j\right\}$, and wages $\left\{w^f\right\}$, such that
  i) each individual chooses consumption and factor supply to maximize utility subject to the budget constraint taking prices as given
  $$\max u^i\left(\left\{c^{ij}\right\}_{j\in\mathcal{J}}, \left\{n^{if,s}\right\}_{f\in\mathcal{F}}\right) \quad \text{s.t.}$$
  $$\sum_j p^j c^{ij} = \sum_j p^j \bar{y}^{ij,s} + \sum_f w^f\left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij}\pi^j,$$
  ii) each technology is operated to maximize profits taking prices as given
  $$\max p^j y^{j,s} - \sum_f w^f n^{jf,d} - \sum_\ell p^\ell x^{j\ell}$$
  iii) and markets clear, that is, resource constraints hold:
  $$y^{j,s} + \sum_i \bar{y}^{ij,s} = \sum_i c^{ij} + \sum_\ell x^{\ell j} \qquad \text{(Resource constraint: goods)}$$
  $$\sum_i n^{if,s} + \sum_i \bar{n}^{if,s} = \sum_j n^{jf,d} \qquad \text{(Resource constraint: factors)}$$

Competitive Equilibrium: Characterization

- Individual optimality:

$$\frac{\partial u^i}{\partial c^{ij}} \leq \lambda^i p^j, \quad \forall i, \forall j \quad \text{and} \quad -\frac{\partial u^i}{\partial n^{if,s}} \geq \lambda^i w^f, \quad \forall i, \forall f,$$

  with equality when $c^{ij} > 0$ and $n^{if,s} > 0$

- Firm optimality:

$$p^j \frac{\partial G^j}{\partial x^{j\ell}} \leq p^\ell, \quad \forall j, \forall \ell \quad \text{and} \quad p^j \frac{\partial G^j}{\partial n^{jf,d}} \leq w^f, \quad \forall j, \forall f,$$

  with equality when $x^{j\ell} > 0$ and $n^{jf,d} > 0$

References I

Dávila, E., and A. Schaab (2024): "Welfare Accounting," *Working Paper*.