Corrective Financial Regulation with Imperfect Instruments*

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Abstract

This paper studies the optimal design of financial regulatory policy when some agents or activities cannot be perfectly regulated. We show that *leakage elasticities* and *Pigovian wedges* are sufficient statistics to account for the marginal welfare impact of imperfect regulatory policies in a large class of environments. We sequentially characterize i) the optimal regulatory policy with unregulated intermediaries, ii) the optimal regulatory policy with regulated and unregulated activities within intermediaries, iii) the value of regulating unregulated intermediaries, and iv) the optimal one-size-fit-all regulation. We quantitatively illustrate our results in an application to shadow banking and show how to reinterpret the results of the existing empirical literature through the lens of our framework.

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1 Introduction

In the aftermath of the 2007-2009 financial crisis, most modern economies have increased and expanded the set of regulations faced by the financial sector, which has spurred heated debates about the unintended consequences of regulation. For instance, the increase in intermediation activities carried outside of the regulated financial sector observed in the US and China in recent years (see e.g., Buchak et al. (2018*b*), Hachem (2018)) is often interpreted as an immediate consequence of tighter financial regulation faced by regulated intermediaries.

In an ideal world, if regulation is desirable, financial regulators would have the ability to regulate every economic decision made by every market participant. However, for different reasons, financial regulation in practice must be conducted with imperfect instruments. In particular, it may be that a given regulator is able to regulate some institutions, but not others. Or it may be that a regulator can regulate some activities of a given intermediary, but not all of them. Alternatively, it may be that a regulator must adopt a one-size-fits-all policy that regulates certain activities but cannot discriminate across different intermediaries. In this paper, we show that the design of optimal regulation in these and related scenarios can be derived from the same set of principles.

We formally study the optimal design of optimal corrective financial regulation in environments in which some agents or activities cannot be perfectly regulated. We initially consider a general model in which multiple intermediaries have access to a rich set of investment and financing opportunities, and can decide to default on their obligations to their creditors ex-post. Importantly, we assume that intermediaries may receive a government bailout ex-post, which is determined under lack of commitment. We begin by assuming that the intermediaries' financing and investment decisions are subject to a potentially rich set of regulations. By sequentially imposing different constraints on the set of regulations, we are able to consider different regulatory scenarios. While it is common to refer to the impact of regulating a given activity on other activities as an unintended consequence, our paper studies how to incorporate those unintended consequences when formulating ex-ante optimal policies.

Our most general result characterizes the marginal welfare impact of varying any given regulatory instrument. It shows that the marginal welfare effects of corrective taxation are fully determined by two sets sufficient sufficient statistics: Leakage elasticities and Pigouvian wedges. The first set of statistics, the leakage elasticities, correspond to the equilibrium responses of financing and investment decisions, both across intermediaries and within intermediaries. The second set of statistics, the Pigouvian wedges, correspond to the difference between the existing corrective regulation that directly affects a given activity and the actual marginal distortion (or externality) generated by that activity. These wedges, which can take positive or negative values, capture the extent to which different activities are regulated too strictly or too leniently for any given set of corrective taxes. We leverage this general characterization to derive our next results.

As an benchmark, we initially derive the first-best solution to the problem of a planner who has access to a full set of instruments. In that case, the Pigou principle applies: the optimal regulation is chosen so that all Pigouvian wedges are exactly equal to zero. An important insight is that leakage elasticities do not form part of the first-best planning solution. In other words, leakage elasticities are only inherently important for corrective regulation in a second-best world, in which the set of regulatory instruments is imperfect. The Pigou principle provides a natural benchmark against which we can compare second-best optimal policies in those environments.

To illustrate the usefulness of our general characterization, we employ it to characterize a range of second-best solutions to widely debated policy problems.

First, we consider a model with unregulated intermediaries. We assume that a policy-maker can freely impose corrective taxes on a regulated segment of intermediaries, but cannot tax another, unregulated segment. This part of our analysis relates to the debate on how commercial banks should be regulated (e.g., via capital and liquidity requirements under Basel III) when the government is (as yet) unable to impose similar requirements on "shadow banks", mutual funds, large insurers, and other financial institutions. In this case, we derive a condition for optimal policy that depends only on the distortions caused by the unregulated segment and the leakage elasticities between regulated and unregulated intermediaries. An intriguing result is that the optimal policy in the regulated segment is commonly *sub-Pigouvian*. Concretely, in existing work that models unregulated intermediaries (e.g., Begenau and Landvoigt, 2018; Bengui and Bianchi, 2018; Chrétien and Lyonnet, 2019), it is typical for the leverage choices of unregulated intermediaries to impose negative externalities, and for unregulated and regulated intermediaries' activities to be gross substitutes. In this setting, and unlike in the first-best benchmark, our formula implies that corrective taxation should stop short of the Pigou principle, and allow private marginal costs to remain below social marginal costs.

Second, we consider unregulated activities *within* intermediaries. For example, this case captures a situtation in which some commercial banks can conduct risky activities "off balance sheet," which are then treated more favorably in regulatory capital requirements. Moreover, most regulatory rules in reality constrain ratios (e.g., the ratio of banks' Tier I capital to risk-weighted assets), while leaving the overall *scale* of the balance sheet as a free variable. In this situation, unregulated and regulated activities (e.g., leverage and the scale of risky investment) are often gross complements (e.g., Dávila and Walther, 2020*b*). Our formula implies that, in these models, optimal policy is *super-Pigouvian*, and features corrective taxes that raise private marginal costs above social marginal costs.

Third, we derive an expression for the marginal welfare benefits of extending regulation to an unregulated segment. This is an important object for current policy proposals that give policy-makers the power to bring non-bank intermediaries into the regulatory perimeter (e.g., Gorton et al., 2010; Adrian and Ashcraft, 2016). We show that the marginal value of such a policy depends not only on the response of the newly regulated intermediary, but also on the substitution effects among all other unregulated segments of the financial system. This additional leakage elasticity potentially reduces the marginal value of new interventions.

Finally, we revisit the analysis of imperfect corrective pricing in (Diamond, 1973), and extend our formula to a case where the government must impose equal taxes on all intermediaries. This characterization can be used to evaluate recent proposals of "activity-based" regulation, and also the effectiveness of monetary policy intervention in controlling unregulated intermediaries' activities (e.g., Stein, 2013; Kress, 2017; Xiao, 2020*a*).

We also provide a explicit illustration of our results in a tractable application of the general model to shadow banking, with an emphasis on connecting leakage elasticities to model primitives. We conclude our analysis by connecting our results with the existing empirical literature. A central implication of our results is that measuring leakage elasticities is critical to assess financial regulatory reforms with imperfect instruments. In this section, we connect this reasoning to existing empirical evidence, which provides direct measurement of the relevant statistics. We revisit empirical work relating to three examples that we considered in Section 3: Optimal regulation with leakage across intermediaries (i.e., unregulated institutions), leakage within intermediaries (i.e., unregulated activities), and the effect of one-size-fits-all policies that effect all intermediaries.

Our paper is most directly related to the growing literature on regulatory arbitrage and shadow banking, which studies the impact of regulatory policies in environments with unregulated financial intermediaries. Pozsar et al. (2010), Gorton et al. (2010), and Claessens et al. (2012) provide a detailed overview of shadow banking institutions, activities, and regulations. Adrian and Ashcraft (2016) and Hachem (2018) provide more recent accounts of the the role of shadow banking in the US and China, respectively.

Within the theoretical literature, Harris, Opp and Opp (2014), Plantin (2015), Farhi and Tirole (2017), Huang (2018), Martinez-Miera and Repullo (2019) study the impact of capital requirements on banking activity and financial stability. Hachem and Song (2017) study how increased liquidity requirements can generate credit booms when banks are heterogeneous. Gennaioli, Shleifer and Vishny (2013), Moreira and Savov (2017), and Ordoñez (2018) develop theories that highlight the fragile nature of shadow banking arrangements. Bengui and Bianchi (2018) provide a theoretical and quantitative macroeconomic analysis of macroprudential policy with imperfect instruments based on a collateral pecuniary externality. Begenau and Landvoigt (2018) provide a quantitative assessment of regulating commercial banks for financial stability and macroeconomic outcomes. See also Dempsey (2020), for a related quantitative assessment. Dávila and Korinek (2018) discuss applications with imperfect instruments in a setup with pecuniary externalities. Korinek (2017) studies optimal corrective regulation in environments with multiple regulators.

There is a growing empirical literature on regulatory arbitrage and shadow banking, which includes the work of Acharya, Schnabl and Suarez (2013), Demyanyk and Loutskina (2016), and Buchak et al. (2018*b*), among others. See also the work of Buchak et al. (2018*a*), who develop a structural quantitative model of lending with regulated and unregulated intermediaries. We explicitly connect our results with this body of work in Section 5.

In our framework, the presence of bailouts — determined ex-post under lack of commitment — provides the rationale for corrective government intervention. By adopting that rationale for regulation, our results are also related to the growing literature on government bailouts, which includes recent contributions by Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Gourinchas and Martin (2017), Cordella, Dell'Ariccia and Marquez 2018, Dávila and Walther (2020*a*) and Dovis and Kirpalani (2020), among others.

Methodologically, our approach develops a sufficient statistic approach to account for the imperfect regulation. Related work in the context of credit markets, banking, and financial intermediation includes Matvos (2013), who uses sufficient statistics to estimate the benefits of contractual completeness, Dávila (2020), who studies the optimal determination of bankruptcy exemptions, Sraer and Thesmar (2018), who show how to produce aggregate estimates from individual firm's experiments, Dávila and Goldstein (2018), who explore the determination of deposit insurance, and Van den Heuvel (2019), who studies the welfare effects of bank liquidity and capital requirements.

At a more abstract level, our results are connected to the literature on public finance that deals with imperfect corrective regulation. In fact, one of our main results revisits the classic principle in Diamond (1973), who studies optimal corrective commodity taxation with imperfect instruments. Other contributions in this literature, often focused on whether indirect regulation is effective or even more desirable than direct regulation, include Green and Sheshinski (1976), Balcer (1980), Wijkander (1985), and Cremer, Gahvari and Ladoux (1998), among others. Corrective regulation is often discussed in the context of environmental policy and congestion; see Bovenberg and Goulder (2002) for a comprehensive review of that literature. Rothschild and Scheuer (2014) and Rothschild and Scheuer (2016) make recent contributions that jointly study optimal taxation with both corrective and redistributive motives with imperfect instruments.

2 Model framework

This section lays out our general framework. We consider an environment in which intermediaries have a rich set of financing and investment decisions that can be subject to a potentially rich set of regulations. Our model is general enough to capture a wide range of scenarios, but sufficiently tractable to yield precise insights and highlight the channels at work. In Section 4, we provide concrete illustrations of our results in a tractable application to shadow banking.

2.1 Environment

There are two dates, indexed by 0 and 1, and a single consumption good (dollar), which serves as numeraire. There are three types of agents: intermediaries and creditors, each in unit measure, and a government, which sets regulatory policy.[1] There is a finite number of intermediary types (intermediaries for short), which we index by $i \in \mathcal{I}$. We index the unit measure of creditors by C. At date 1, there is a continuum of possible states of nature $s \in [\underline{s}, \overline{s}]$, with cdf $F(s)$, where $\underline{s} \geq 0$.

At date 0, intermediaries can invest in N_k different forms of capital investment and have access to N_b sources of financing. At date 1, intermediaries receive the return on their investments, potentially receive a bailout transfer from the government, and decide whether to repay their financial obligations, as we describe next. Creditors fund intermediaries at date 0. We assume that the profits from capital production accrue to intermediaries.

Intermediaries. Intermediaries have well-behaved preferences of the form

$$u_0^i\left(c_0^i\right) + \beta^i \int u_1^i\left(c_1^i(s)\right) dF(s), \tag{1}$$

where $u_0^i(\cdot)$ and $u_1^i(\cdot)$ denote the flow utility of intermediary i, $\beta^i \in (0,1]$ denotes the discount factor, and where c_0^i and $c_1^i(s)$ denote intermediaries' consumption at each date/state.[2] Intermediary i faces the following budget constraints:

$$c_0^i \leq n_0^i + Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) - \boldsymbol{\tau}_b^i \cdot \boldsymbol{b}^i - \left(\boldsymbol{p}_k + \boldsymbol{\tau}_k^i\right) \cdot \boldsymbol{k}^i - \Upsilon^i\left(\boldsymbol{k}^i\right) + T_0^i \tag{2}$$

$$c_1^i(s) \leq n_1^i(s) + \max\left\{v^i\left(\boldsymbol{k}^i, s\right) + t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right), 0\right\}, \quad \forall s. \tag{3}$$

[1]The notion of intermediaries in this paper should be broadly understood. We could have referred to intermediaries as experts, investors, or entrepreneurs. We use the terminology "intermediary" because it maps better to the set of agents that face financial regulations in practice.

[2]We denote vectors with bold font, as in $\boldsymbol{b}^i$ or $\boldsymbol{\tau}_b^i$, and dot products with $\cdot$, as in $\boldsymbol{\tau}_b^i \cdot \boldsymbol{b}^i$.

At date 0, an intermediary i is initially endowed with n_0^i dollars. An intermediary makes N_b financing decisions via the $N_b \times 1$ vector $\boldsymbol{b}^i$, and makes N_k investment decisions via the $N_k \times 1$ vector $\boldsymbol{k}^i$. We denote the total amount of financing raised by intermediary i by $Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right)$, whose determination in equilibrium is described below. Intermediaries can purchase capital goods at a price vector $\boldsymbol{p}_k$, and face investment adjustment costs given by $\Upsilon^i\left(\boldsymbol{k}^i\right)$, where $\Upsilon^i(\cdot)$ is an increasing and convex function. Intermediaries can receive a lump-sum transfer $T_0^i \geq 0$ at date 0.

Importantly, we allow for (potentially) intermediary-specific taxes/subsidies on financing ($N_b \times 1$ vector $\boldsymbol{\tau}_b^i$) and investment ($N_k \times 1$ vector $\boldsymbol{\tau}_k^i$) decisions. In our regulating, regulating $\boldsymbol{b}^i$ through $\boldsymbol{\tau}_b^i$ can be interpreted as setting capital (liability-side) regulation, while regulating $\boldsymbol{k}^i$ through $\boldsymbol{\tau}_k^i$ can be interpreted as setting liquidity (asset-side) regulation. By imposing constraints on the permitted values of of $\boldsymbol{\tau}_b^i$ and $\boldsymbol{\tau}_k^i$, we will be able to consider different regulatory scenarios in Section 3.

At date 1, the capital investments of intermediaries yield $v^i\left(\boldsymbol{k}^i, s\right) \geq 0$ in state s. After the state s is realized, the government makes a bailout transfer $t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) \geq 0$ to intermediaries. Because our results do not hinge on the exact form of the bailout, we take the bailout policy $t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)$ as a primitive of the model. This approach is fully consistent with $t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)$ being chosen at date 1 by a government that lacks commitment ex-post.[3] The liabilities of intermediary i in state s, given by $\Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)$, are potentially defaultable. If intermediaries default, they discharge their liabilities, but lose the return on their investments and any bailout funds. Therefore, intermediaries find it optimal to default when $v^i\left(\boldsymbol{k}^i, s\right) + t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) > 0$ and to repay otherwise. For simplicity, we assume that creditors have no recourse to intermediaries' endowments, given by $n_1^i(s)$, although our results extend with minimal modifications to environments with alternative assumptions on recourse.

Creditors. Creditors have well-behaved preferences of the form

$$u_0^C\left(c_0^C\right) + \beta^C \int u_1^C\left(c_1^C(s)\right) dF(s), \tag{4}$$

where $u_0^C(\cdot)$ and $u_1^C(\cdot)$ denote the creditors' flow utility, $\beta^C \in (0,1]$ denotes their discount factor, and where c_0^C and $c_1^C(s)$ denote creditors' consumption at each date/state. Creditors

[3] The desire to bail out financial intermediaries ex-post under lack of commitment has been widely explored in the literature, as we describe in details in the Introduction.

face the following budget constraints:

$$c_0^C \leq n_0^C - \sum_{i \in \mathcal{I}} d_i^C Q^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i \right) + \Pi_k \left(\boldsymbol{p}_k \right) \tag{5}$$

$$c_1^C (s) \leq n_1^C (s) + \sum_{i \in \mathcal{I}} d_i^C \tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right) - \kappa \left(T_1 (s), s \right), \quad \forall s. \tag{6}$$

At date 0, creditors are initially endowed with n_0^C dollars. They choose to finance a share d_i^C of each intermediary i financing needs (in equilibrium, as we explain below, $d_1^C = 1$). We assume without consequence for our results that the profits from capital production $\Pi_k (\boldsymbol{p}_k)$ accrue to creditors. Capital producers choose capital supply $\boldsymbol{k}^S$ to solve the problem

$$\Pi_k (\boldsymbol{p}_k) = \max_{\boldsymbol{k}^S} \boldsymbol{p}_k \cdot \boldsymbol{k}^S - \Psi \left(\boldsymbol{k}^S \right), \tag{7}$$

where $\Psi (\boldsymbol{k})$ is a convex cost of production.

At date 1, creditors have an endowment of $n_1^i (s)$ and receive a per-share repayment $\tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$ from creditors in each state. The repayment term $\tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$ accounts for the possibility that an intermediary i may default. In particular, when intermediary i repays, then $\tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right) = \Omega^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$. However, when intermediary i decides to default, $\tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$ would account for the recovery value to creditors. This recovery value could account for default-induced deadweight losses. Finally, $T_1 (s)$ corresponds to the revenue raised by the government to finance bailout transfers to intermediaries and $\kappa (T_1 (s), s)$ denotes the total tax required to raise this revenue. We assume that $\kappa (T)$ is convex, and allow for deadweight fiscal costs, so $\kappa (T) - T > 0$, $\forall T > 0$. By indexing $\kappa (\cdot)$ by s, we allow for the fiscal cost to vary with the state of the economy.

To simplify the exposition, we have chosen an environment in which intermediaries and creditors are different along two significant dimensions. First, creditors do not have access to investment opportunities between dates 0 and 1. Second, creditors do not have a commitment problem. This distinction, which helps the exposition, is somewhat artificial: It is possible to generate an equilibrium of the form we consider in this paper in an environment in which agents of the same type sort into being intermediaries and creditors.

Government. The government intervenes in this economy at dates 0 and 1. At date 0, the budget constraint of the government is given by

$$\sum_{i \in \mathcal{I}} T_0^i = \sum_{i \in \mathcal{I}} \left(\boldsymbol{\tau}_b^i \cdot \boldsymbol{b}^i + \boldsymbol{\tau}_k^i \cdot \boldsymbol{k}^i \right). \tag{8}$$

As shown below, our results are valid under arbitrary rebate policies, so we only have to characterize the sum $\sum_{i \in \mathcal{I}} T_0^i$. At date 1, the budget constraint of the government is given by

$$T_1(s) = \sum_{i \in \mathcal{I}} t^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right). \tag{9}$$

Our main results consider regulatory scenarios that can be modeled as constraints on the set of instruments $\boldsymbol{\tau}_b^i$ and $\boldsymbol{\tau}_k^i$ available to the government. For instance, the first-best policy, which we characterize below, allows the government to set every element of $\boldsymbol{\tau}_b^i$ and $\boldsymbol{\tau}_k^i$ freely. Alternatively, we consider scenarios in which the government i) regulates a single financing or investment activity by a given intermediary i or ii) is forced to impose the same regulation among different intermediaries.

Equilibrium definition. An *equilibrium* with corrective taxes $\{\boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i, T_0^i\}_{i \in \mathcal{I}}$ and ex-post bailout transfers $\left\{ \left\{ t^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right) \right\}_{i \in \mathcal{I}}, T_1(s) \right\}$ consists of consumption bundles $\{c_0^i, c_1^i(s)\}_{i \in \mathcal{I}}$ and $\left\{ c_0^C, c_1^C(s) \right\}$, intermediaries' financing and investment decisions $\left\{ \boldsymbol{b}^i, \boldsymbol{k}^i \right\}_{i \in \mathcal{I}}$, creditors' financing decisions $\left\{ d_i^C \right\}_{i \in \mathcal{I}}$, capital supply $\boldsymbol{k}^S$, financing schedules $\left\{ Q^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i \right) \right\}_{i \in \mathcal{I}}$, intermediary liabilities $\Omega^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$, creditors' repayment $\tilde{\Omega}^i \left(\boldsymbol{b}^i, \boldsymbol{k}^i, s \right)$ given intermediaries default decisions, and capital prices $\boldsymbol{p}_k$, such that i) intermediaries maximize utility, Equation (1), subject to budget constraints (2) and (3), taking capital prices as given but internalizing how their actions affect the repayment of creditors, ii) creditors maximize utility, Equation (4), subject to budget constraints (5) and (6), iii) capital is produced optimally, as in (7), iv) the government's budget constraints (8) and (9) are satisfied, and v) the market clearing conditions for securities and capital, respectively given by

$$d_i^C = 1, \ \forall i \quad \text{and} \quad \sum_{i \in \mathcal{I}} \boldsymbol{k}^i = \boldsymbol{k}^S, \tag{10}$$

are satisfied.

We would like to make two final remarks on the environment. First, it is straightforward to extend our model to include non-pecuniary benefits that accrue to creditors after making some specific financing choices or holding some particular security. This extension would account for the notion that some liabilities are special and are priced above and beyond their pecuniary properties – see e.g., Sunderam (2015) or Begenau and Landvoigt (2018). Our main results remain unchanged in this case.

Second, our environment encompasses scenarios with both complete or incomplete markets, subject to our commitment assumptions. Because we are in a two-date environment without price-dependent constraints, all distributive pecuniary externalities add up to zero — as shown in Lemma 1 below. Considering additional periods and/or price-dependent borrowing constraints would introduce additional rationales for corrective policies. Our insights about corrective regulation remain valid in that case with the pecuniary rationales for regulation entering in the same way the bailout distortions do. The same logic applies to other form of ad-hoc externalities, as we show in the Appendix.[4]

2.2 Equilibrium

The equilibrium of the model can be characterized backwards. At date 1, intermediaries make default decisions. It follows directly from Equation (3) that the set of states in which intermediaries default at date 1 can be characterized as follows

$$\begin{aligned} &\text{if } \boldsymbol{v}^i\left(\boldsymbol{k}^i, s\right) + t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) < \Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right), \quad \text{Default } (\mathcal{D}) \\ &\text{if } \boldsymbol{v}^i\left(\boldsymbol{k}^i, s\right) + t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) \geq \Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right), \quad \text{No Default } (\mathcal{N}) \end{aligned} \tag{11}$$

and will repay otherwise. Given the ex-post default decisions by intermediaries, we can express the optimality conditions for creditors' financing decisions by

$$Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) = \int m^C(s)\,\tilde{\Omega}^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) dF(s), \tag{12}$$

where $m^C(s) = \frac{\beta^C u^{C\prime}\left(c_1^C(s)\right)}{u^{C\prime}\left(c_0^C\right)}$ denotes the stochastic discount factor of creditors. Equation (12) defines a supply of credit (credit surface) that determines financing conditions.

The optimality conditions that determine the optimal supply of capital are

$$\boldsymbol{p}_k = \Psi'\left(\boldsymbol{k}^S\right).$$

Finally, slightly abusing notation, we can express the optimality conditions of intermediaries for $\boldsymbol{b}^i$ and $\boldsymbol{k}^i$ as follows:

$$\frac{\partial Q^i}{\partial \boldsymbol{b}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) - \boldsymbol{\tau}_b^i = \int_{\mathcal{N}} m^i(s)\left(\frac{\partial \Omega^i}{\partial \boldsymbol{b}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \frac{\partial t^i}{\partial \boldsymbol{b}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)\right) dF(s) \tag{13}$$

$$\frac{\partial Q^i}{\partial \boldsymbol{k}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) - \left(\boldsymbol{p}_k + \boldsymbol{\tau}_k^i + \frac{d\Upsilon^i}{d\boldsymbol{k}^i}\left(\boldsymbol{k}^i\right)\right) = \int_{\mathcal{N}} m^i(s)\left(\frac{\partial v^i}{\partial \boldsymbol{k}^i}\left(\boldsymbol{k}^i, s\right) + \frac{\partial \Omega^i}{\partial \boldsymbol{k}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \frac{\partial t^i}{\partial \boldsymbol{k}^i}\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)\right) dF(s), \tag{14}$$

where $m^i(s) = \frac{\beta^i u^{i\prime}\left(c_1^i(s)\right)}{u^{i\prime}\left(c_0^i\right)}$ denotes the stochastic discount factor of intermediary i and where $\mathcal{N}$ denotes the no default region, as defined in Equation (11).

[4]There scope to extend our results to environments with strategic intermediaries and imperfect competition, as in Corbae and D'Erasmo (2010); Corbae and Levine (2018, 2019) and Dávila and Walther (2020*a*).

3 Optimal corrective regulation

We are now ready to explore the form of the optimal corrective regulation. We allow the planner to choose the corrective tax regime described by $\{\boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i, T_0^i\}_{i\in\mathcal{I}}$ in different scenarios. As described above, the planner takes as given the date 1 bailout and taxation policies, $\left\{t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right)\right\}_{i\in\mathcal{I}}$ and $T_1(s)$, which is consistent with these policies chosen ex-post without commitment.

We denote the indirect utility of intermediary i by

$$\tilde{V}^i\left(Q^i, \boldsymbol{p}_k, \boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i, T_0^i\right).$$

This function depends on the financing schedule faced by intermediary i, Q^i, the prices of capital, $\boldsymbol{p}_k$, the corrective taxes on financing and investment decisions, $\boldsymbol{\tau}_b^i$ and $\boldsymbol{\tau}_k^i$, and on the lump-sum transfer T_0^i. We denote the indirect utility of creditors by

$$\tilde{V}^C\left(\left\{Q^i\right\}_{i\in\mathcal{I}}, \Pi_k, \{T_1(s)\}_s\right).$$

This function depends on the financing schedules of all intermediaries at date 0, $\{Q^i\}_{i\in\mathcal{I}}$, the profits earned from capital production, Π_k, and the bailout taxes at date 1 required in each state s.

In order to abstract from redistributional concerns and focus on aggregate efficiency, we compute the welfare change of social welfare in monetary units. That is, when computing the marginal welfare effect of a change in a given variable x, we respectively define $\frac{dV^i}{dx} = \frac{\frac{d\tilde{V}^i}{dx}}{\tilde{\lambda}_0^i}$ for intermediary i and $\frac{dV^C}{dx} = \frac{\frac{d\tilde{V}^C}{dx}}{\tilde{\lambda}_0^C}$ for creditors, where $\tilde{\lambda}_0^i$ denotes the social marginal value of a dollar at date 0 in the hands of intermediary i and $\tilde{\lambda}_0^C$ denotes the social marginal value of a dollar at date 0 in the hands of creditors. We use the same notation for partial derivatives, that is, $\frac{\partial V^i}{\partial x} = \frac{\frac{\partial\tilde{V}^i}{\partial x}}{\tilde{\lambda}_0^i}$ and $\frac{\partial V^C}{\partial x} = \frac{\frac{\partial\tilde{V}^C}{\partial x}}{\tilde{\lambda}_0^C}$. Consequently, the aggregate welfare effect — when measured in dollars — of a change in a given variable x, which we denote by $\frac{dW}{dx}$ or $\frac{\partial W}{\partial x}$, is simply given by

$$\frac{dW}{dx} = \sum_{i\in\mathcal{I}}\frac{dV^i}{dx} + \frac{dV^C}{dx}.$$

This is a natural approach when dealing with corrective problems in non-quasilinear economies. As we explain in the Appendix, this approach can be interpreted as selecting an equal-weighted set of "generalized social welfare weights", using the approach laid out in Saez and Stantcheva (2016).

We make three further remarks on notation. First, we adopt the same notational conventions for vector-valued derivatives. For example, the total effect of a marginal change in the tax vector $\boldsymbol{\tau}_b^i$ on welfare (in dollars) is denoted $\frac{dW}{d\boldsymbol{\tau}_b^i}$, and is a vector with the same dimensions as $\boldsymbol{\tau}_b^i$, and the partial effect of a change in capital prices is $\frac{\partial V^i}{\partial \boldsymbol{p}_k}$. Second, we use derivatives of vector-valued functions where appropriate. For example, the total effect of a change in taxes $\boldsymbol{\tau}_b^i$ on the security issuance vector $\boldsymbol{b}^j$ of intermediary j is $\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i}$, and it denotes a matrix in which each column represents the derivatives of one element of $\boldsymbol{b}^j$ with respect to taxes.

Finally, we use dot product notation to modulate these derivatives by the appropriate prices and taxes. For instance, we write $\boldsymbol{\tau}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} \equiv \sum_k \tau_b^{j,k} \frac{db^{j,k}}{d\boldsymbol{\tau}_b^i}$, which yields a vector with the same length as $\boldsymbol{\tau}_b^i$.

3.1 Marginal Welfare Effects of Corrective Taxation

When computing the welfare impact of policy, the planner must take into account general equilibrium effects, in particular, the impact of policy on prices. In Lemma 1, we show that the welfare impact of changes in equilibrium prices is zero-sum on aggregate — using the language of Dávila and Korinek (2018), the distributive pecuniary impact of a policy is zero in the aggregate. This result simplifies the characterization of the aggregate marginal impact of policies.

Lemma 1. *[Zero-sum distributive pecuniary welfare effects] The aggregate marginal welfare effect — measured in dollars — of a change in market prices is zero. In particular, in any equilibrium:*

1. *The welfare effect of a change in the financing schedule $Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right)$ faced by any intermediary $i \in \mathcal{I}$ is given by*

$$\frac{\partial W}{\partial Q^i} = \frac{\partial V^C}{\partial Q^i} + \sum_{i \in \mathcal{I}} \frac{\partial V^i}{\partial Q^i} = 0. \tag{15}$$

2. *The welfare effect of a change in capital prices $\boldsymbol{p}_k$ is given by*

$$\frac{\partial W}{\partial \boldsymbol{p}_k} = \frac{\partial V^C}{\partial \Pi_k} \frac{\partial \Pi_k}{\partial \boldsymbol{p}_k} + \sum_{i \in \mathcal{I}} \frac{\partial V^i}{\partial \boldsymbol{p}_k} = 0. \tag{16}$$

Given Lemma 1, we can characterize the aggregate marginal welfare effects of any corrective intervention. We define $\mu(s)$ as the marginal welfare cost, expressed in units of date 0 dollars, of raising one dollar of fiscal revenue from creditors at date 1 in state s, that is,

$$\mu(s) = \kappa'(T_1(s))\, m^C(s) \tag{17}$$

where $m^C(s) = \frac{\beta^C u^{C\prime}\left(c_1^C(s)\right)}{u^{C\prime}\left(c_0^C\right)}$ denotes the stochastic discount factor of creditors. Note that when there are no deadweight losses from taxation, $\kappa'(T_1(s)) = 1$, so $\mu(s)$ simply corresponds to the stochastic discount factor of creditors in that case.

Proposition 1 introduces our most general result. It characterizes the marginal welfare impact of changing a given regulatory instrument, either a member of $\boldsymbol{\tau}_b^i$ or $\boldsymbol{\tau}_k^i$. Note that both $\frac{dW}{d\boldsymbol{\tau}_b^i}$ and $\frac{dW}{d\boldsymbol{\tau}_k^i}$ are vector-valued derivatives.

Proposition 1. *[Marginal Welfare Effect of Corrective Taxes: Leakage Elasticities and Pigouvian Wedges] The marginal welfare effects of raising financing taxes and investment taxes on intermediary $i \in \mathcal{I}$ are respectively given by*

$$\frac{dW}{d\boldsymbol{\tau}_b^i} = \sum_{j \in \mathcal{I}} \left[\left(\boldsymbol{\tau}_b^j - \boldsymbol{\delta}_b^j\right) \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} + \left(\boldsymbol{\tau}_k^j - \boldsymbol{\delta}_k^j\right) \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i} \right], \text{ and} \tag{18}$$

$$\frac{dW}{d\boldsymbol{\tau}_k^i} = \sum_{j \in \mathcal{I}} \left[\left(\boldsymbol{\tau}_b^j - \boldsymbol{\delta}_b^j\right) \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_k^i} + \left(\boldsymbol{\tau}_k^j - \boldsymbol{\delta}_k^j\right) \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_k^i} \right]. \tag{19}$$

where $\boldsymbol{\delta}_b^j$ and $\boldsymbol{\delta}_k^j$ are the marginal bailout externalities (Pigouvian distortions), defined by

$$\boldsymbol{\delta}_b^j = \mathbb{E}\left[\mu(s) \frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{b}^j}\right], \quad \boldsymbol{\delta}_k^j = \mathbb{E}\left[\mu(s) \frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{k}^j}\right]. \tag{20}$$

Proposition 1 highlights that the marginal welfare effects of corrective taxation are fully determined by two sets sufficient sufficient statistics: leakage elasticities and Pigouvian wedges.

The first set of statistics are the *leakage elasticities*, such as the general equilibrium responses $\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i}$ and $\frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i}$ of intermediary j's financing and investment to a tax on intermediary i. Notice that the leakage elasticities both across intermediaries (e.g., $\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i}$) and across activities within the same intermediary (e.g., $\frac{d\boldsymbol{k}^i}{d\boldsymbol{\tau}_b^i}$) are relevant. These elasticities measure how taxation of one activity affects other equilibrium quantities. For example, the role of intermediary j in Equation (18) depends on whether financing choices are gross substitutes ($\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} > 0$) or gross complements ($\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} < 0$) across intermediaries. We demonstrate below that either the gross substitutes or the gross complements cases can be relevant for second-best policy in different practical scenarios that have been considered in the existing literature. The role of leakage elasticities in our analysis also sheds light on recent empirical evidence, which we discuss in detail in Section 5.

The second set of statistics are the *Pigouvian wedges* between corrective taxes and marginal distortions, such as $\boldsymbol{\tau}_b^i - \boldsymbol{\delta}_b^j$ and $\boldsymbol{\tau}_k^i - \boldsymbol{\delta}_k^i$. It is convenient to define the financing and investment wedges more concisely as

$$\boldsymbol{\omega}_b^i = \boldsymbol{\tau}_b^i - \boldsymbol{\delta}_b^i, \tag{21}$$
$$\boldsymbol{\omega}_k^i = \boldsymbol{\tau}_k^i - \boldsymbol{\delta}_k^i, \tag{22}$$

For any given set of corrective taxes, these wedges capture the extent to which different activities are regulated too strictly or too leniently. For example, if $\boldsymbol{\omega}_b^j < 0$, then the corrective tax on security issuance by intermediary j is smaller than the marginal distortion that this activity creates. Negative wedges therefore imply that private marginal costs are smaller than social marginal costs, while positive wedges imply that private marginal costs exceed social marginal costs.[5] In terms of Pigouvian wedges, the marginal welfare effects in Proposition 1 can be expressed as

$$\frac{dW}{d\boldsymbol{\tau}_b^i} = \sum_{j\in\mathcal{I}} \left(\boldsymbol{\omega}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} + \boldsymbol{\omega}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i} \right)$$
$$\frac{dW}{d\boldsymbol{\tau}_b^k} = \sum_{j\in\mathcal{I}} \left(\boldsymbol{\omega}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^k} + \boldsymbol{\omega}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^k} \right).$$

In common with other studies of second-best corrective policy (e.g., Diamond, 1973; Farhi and Werning, 2016), our formulae show that the key welfare effects are weighted averages of wedges, and in particular, the relevant weights are leakage elasticities in our model. Intuitively, welfare increases if a policy reform discourages activities (e.g., $\frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} < 0$) that are currently regulated too leniently (e.g., $\boldsymbol{\omega}_b^j < 0$).

The marginal welfare changes presented in Proposition 1 are useful for analyzing a wide range of environments in which financial policy is forced to operate with imperfect instruments. In the remainder of this section, we consider a series of scenarios that relate to the existing literature in macroeconomics and finance. We begin with the benchmark of first-best policy.

3.2 The first-best benchmark

In order to obtain the first-best policy, consider a social planner who can set arbitrary corrective taxes $\boldsymbol{\tau}_b^i$ and $\boldsymbol{\tau}_k^i$ for all intermediaries $i \in \mathcal{I}$. Proposition 1 immediately implies a standard characterization of optimal policy:

[5] Clearly, all wedges are be set to zero under first best regulation, which we characterize in more detail below, but may not be zero when the planner has imperfect instruments.

Proposition 2. *[First-Best Policy / Pigou Principle] If the planner can choose taxes* $\boldsymbol{\tau}_b^i$ *and* $\boldsymbol{\tau}_k^i$ *for all intermediaries* $i \in \mathcal{I}$ *without constraints, then any optimal policy satisfies*

$$\boldsymbol{\omega}_b^i = 0 \iff \boldsymbol{\tau}_b^i = \boldsymbol{\delta}_b^i$$
$$\boldsymbol{\omega}_k^i = 0 \iff \boldsymbol{\tau}_k^i = \boldsymbol{\delta}_k^i.$$

This is an instance of the Pigou principle (i.e., the "polluter pays"). The first-best regulation on intermediaries is set to align private and social marginal costs across all activities. In terms of the Pigouvian wedges defined in Equations 21 and 22 above, all wedges are set to zero. Note that Proposition 2 implies that the economy without bailouts is constrained efficient, while introducing bailouts implies that both investment and financing decisions must be regulated.

In the following subsections, we consider scenarios where the planner operates under exogenous constraints on corrective taxation. The Pigou principle provides a natural benchmark against which we can evaluating second-best optimal policies in those environments.

3.3 Corrective policy with unregulated intermediaries

We use Proposition 1 to consider the benefits of imposing higher corrective taxes on a regulated intermediary when there are also unregulated activities in the economy. Assume that only intermediaries $i \in \mathcal{I}^\star \subset \mathcal{I}$ are regulated. The planner can freely impose corrective taxes for one segment of intermediaries $i \in \mathcal{I}^\star$, but is constrained to set $\boldsymbol{\tau}_b^j = \boldsymbol{\tau}_k^j = 0$ for the remaining, unregulated segment of intermediaries $j \notin \mathcal{I}^\star$. Notice that the level of activity of these two segments is determined in general equilibrium. Indeed, both regulated and unregulated intermediaries compete for financing and for capital investment in our environment.

3.3.1 Optimal regulation with unregulated intermediaries

We can use Proposition 1 to characterize the first-order condition for optimal corrective taxes in the regulated segment, which is

$$\frac{dW}{d\boldsymbol{\tau}_b^i} = 0 \quad \text{and} \quad \frac{dW}{d\boldsymbol{\tau}_k^i} = 0, \forall i \in \mathcal{I}^\star.$$

In this scenario, it is instructive to consider a simple variational argument when computing optimal taxes. Concretely, to characterize optimal policy, we consider small tax reforms of the following kind: The planner changes corrective taxes so that, for some regulated intermediary $i \in \mathcal{I}^\star$, its choice of $\boldsymbol{b}^i$ increases by a marginal unit, while all other choices of *regulated* intermediaries remain constant, but the choices in the *unregulated* segment may adjust. If the policy is optimal, then this marginal reform must not improve welfare.

Formally, we define the associated general equilibrium responses as follows:

Definition 1. In any equilibrium with taxes, consider a marginal tax reform $\{d\boldsymbol{\tau}_b^i, d\boldsymbol{\tau}_k^i\}_{i\in\mathcal{I}^\star}$, in response to which *(i)* a single intermediary $i \in \mathcal{I}^\star$ increases her equilibrium choice of security issuance $\boldsymbol{b}^i$ by a marginal unit, while *(ii)* her equilibrium choice $\boldsymbol{k}^i$ of capital investment is unchanged, and *(iii)* the equilibrium choices $\left\{\boldsymbol{b}^{i'}, \boldsymbol{k}^{i'}\right\}_{i'\in\mathcal{I}^\star\setminus\{i\}}$ of all other regulated intermediaries are unchanged.[6] The *general equilibrium response* to this tax reform is defined, for every unregulated intermediary $j \notin \mathcal{I}^\star$, as $\frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i}$ and $\frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}$.

Similarly, consider a marginal tax reform $\{d\boldsymbol{\tau}_b^i, d\boldsymbol{\tau}_k^i\}_{i\in\mathcal{I}^\star}$, in response to which *(i)* a single intermediary $i \in \mathcal{I}^\star$ increases her equilibrium choice of capital investment issuance $\boldsymbol{k}^i$ by a marginal unit, while *(ii)* her equilibrium choice $\boldsymbol{b}^i$ of securities issuance is unchanged, and *(iii)* the equilibrium choices $\left\{\boldsymbol{b}^{i'}, \boldsymbol{k}^{i'}\right\}_{i'\in\mathcal{I}^\star\setminus\{i\}}$ of all other regulated intermediaries are unchanged. The *general equilibrium response* to this tax reform is defined, for every unregulated intermediary $j \notin \mathcal{I}^\star$, as $\frac{d\boldsymbol{b}^j}{d\boldsymbol{k}^i}$ and $\frac{d\boldsymbol{k}^j}{d\boldsymbol{k}^i}$.

Using this notion of local tax reform in conjunction with Proposition 1, we characterize optimal regulation as follows:

Proposition 3. *[Regulation with Unregulated Intermediaries] If the planner is constrained* $\boldsymbol{\tau}_b^i = \boldsymbol{\tau}_k^i = 0$ *for* $i \notin \mathcal{I}^\star \subset \mathcal{I}$*, then any optimal policy satisfies*

$$\boldsymbol{\omega}_b^i = \boldsymbol{\tau}_b^i - \boldsymbol{\delta}_b^i = \sum_{j\notin\mathcal{I}^\star}\left(\boldsymbol{\delta}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i} + \boldsymbol{\delta}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}\right), \tag{23}$$

and

$$\boldsymbol{\omega}_k^i = \boldsymbol{\tau}_k^i - \boldsymbol{\delta}_k^i = \sum_{j\notin\mathcal{I}^\star}\left(\boldsymbol{\delta}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{k}^i} + \boldsymbol{\delta}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{k}^i}\right) \tag{24}$$

for all $i \in \mathcal{I}^\star$.

Proposition 3 illustrates the impact of unregulated intermediaries on the optimal policy in the regulated sector. Equation (23) gives a necessary condition for optimal taxes on security issuance by a regulated intermediary $i \in \mathcal{I}^\star$. The left-hand side is the wedge between the tax and marginal distortion by intermediary i itself, which would be zero in a first-best scenario. The left-hand side shows that this wedge is optimally set to a weighted sum of the Pigouvian

[6] In terms of our notation for general equilibrium effects, the required tax reform is obtained by solving the following equations for all $i' \in \mathcal{I}^\star$:

$$\sum_{m\in\mathcal{I}^\star}\left(\frac{d\boldsymbol{b}^{i'}}{d\boldsymbol{\tau}_b^m} + \frac{d\boldsymbol{b}^{i'}}{d\boldsymbol{\tau}_k^m}\right) = 1\left\{i' = i\right\}, \quad \sum_{m\in\mathcal{I}^\star}\left(\frac{d\boldsymbol{k}^{i'}}{d\boldsymbol{\tau}_b^m} + \frac{d\boldsymbol{k}^{i'}}{d\boldsymbol{\tau}_k^m}\right) = 0$$

distortions $\boldsymbol{\delta}_b^j$ and $\boldsymbol{\delta}_k^j$ caused by unregulated intermediaries. The weights in this sum are the leakage elasticities $\frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i}$ and $\frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}$. These weights are negative when there is substitutability between intermediaries, thus reducing the optimal tax on the regulated sector.

3.3.2 Underregulation versus cracking down on the regulated

Proposition 3 immediately implies is that Pigouvian wedges are *negative* in the regulated sector under two conditions, namely, that (i) the unregulated sector causes positive fiscal distortions, and (ii) regulated and unregulated activities are gross substitutes. These two conditions are satisfied in a range of models that consider some degree of regulatory arbitrage between traditional banks and "shadow banks" (e.g., (e.g., Begenau and Landvoigt, 2018; Chrétien and Lyonnet, 2019; Bengui and Bianchi, 2018). In this case, the optimal taxes in the regulated sector are lower than the Pigou principle implies. Underregulation, in the sense that social marginal costs of intermediaries' activities exceeds private ones, therefore spills over from the unregulated to the regulated sector.

At first glance, this result appears to contradict the common intuition that policy should remain robust in, or "crack down" on, the regulated sector, with the objective of keeping the system as a whole safe when there are leakages. This rationale is evident in the policy response to the crisis of 2008, which was arguably amplified by systemic risks in the unregulated segment of the US financial system, but lead to substantial tightening of capital and liquidity regulation in the regulated segment. By contrast, our arguments above suggest that optimal policy is always sub-Pigouvian when there is an unregulated segment that acts as a substitute.

The friction between these two points of view is easy to resolve in our analysis. Indeed, an important insight is that one should think differently about taxes and wedges. We show that, with substitutability in the unregulated sector, it is optimal to under-regulate across the board, in the sense of negative *wedges* in Equation (23). At the same time, the level of corrective *taxes* might rise with imperfect enforcement, relative to first best. To see this, consider the case with fixed k and note that the planner sets

$$\begin{aligned}\boldsymbol{\tau}_b^i &= \mathbb{E}\left[\mu(s)\frac{\partial t^j\left(\boldsymbol{b}^j,\boldsymbol{k}^j,s\right)}{\partial \boldsymbol{b}^j}\right] + \sum_{j\notin\mathcal{I}^\star}\left(\boldsymbol{\delta}_b^j\cdot\frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i} + \boldsymbol{\delta}_k^j\cdot\frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}\right)\\ &= \mathbb{E}\left[\kappa'\left(T_1(s)\right)\beta^i\frac{u_1^{C\prime}\left(c_1^C(s)\right)}{u_0^{C\prime}\left(c_0^C\right)}\frac{\partial t^j\left(\boldsymbol{b}^j,\boldsymbol{k}^j,s\right)}{\partial \boldsymbol{b}^j}\right] + \sum_{j\notin\mathcal{I}^\star}\left(\boldsymbol{\delta}_b^j\cdot\frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i} + \boldsymbol{\delta}_k^j\cdot\frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}\right),\end{aligned} \tag{25}$$

where we have used the definition of the marginal fiscal cost $\mu(s)$. In the case with substitutes, the second term in this expression reduces taxes relative to first best. However, consider the first term, in particular $\kappa'(T_1(s))$. If the marginal fiscal burden $\kappa'(\cdot)$ rises sufficiently in response to unregulated activity, then the second-best tax on regulated intermediaries can be higher than the first best. More explicitly, suppose we make the leakage problem more severe by granting a small subsidy (i.e., a tax $d\tau_j < 0$) for security issuance to some unregulated intermediary $j \notin \mathcal{I}^\star$. The effect on $\kappa'(\cdot)$ (for a given state s) is

$$\begin{aligned}\frac{d}{d\tau^j}\left\{\kappa'\left(T_1\left(s\right)\right)\right\} &= \kappa''\left(T_1\left(s\right)\right)\frac{d}{d\tau^j}\left\{\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right\} \\ &= \kappa''\left(T_1\left(s\right)\right)\sum_{i\in\mathcal{I}}\underbrace{\frac{\partial t^i\left(b^i, s\right)}{\partial b^i}}_{\text{bailout response}}\cdot\underbrace{\frac{db^i}{d\tau^j}}_{\text{GE response}}\end{aligned}$$

The general equilibrium response to the subsidy tends to be positive for intermediary j receiving the subsidy, and the bailout response tends to be negative, in natural environments with unregulated intermediaries (see, for example, our applications in Section 4). Thus, the overall effect of more severe leakage on the marginal fiscal burden can be positive, which increases the optimal tax on regulated intermediaries in Equation (25). Intuitively, the presence of leakage increases the overall risk of bailouts, which raises the marginal costs of providing bailouts to any regulated entity, thus justifying a larger corrective tax.

3.4 Regulated and unregulated activities within intermediaries

Our analysis also sheds light on situations where the same intermediary can engage in both regulated and unregulated activities. One example of this scenario is off-balance sheet instruments such as Special Purpose Vehicles that are subsidiaries of regulated intermediaries (e.g., Gorton and Souleles, 2007). Another related practice is for regulated intermediaries to manage their investments and internal models so as to minimize the "risk weights" attached to their assets (e.g., Mariathasan and Merrouche, 2014; Behn, Haselmann and Vig, 2016). The latter practice generates unregulated activity because intermediaries can choose among different activities that lead to the same overall taxation via capital charges.

To study this scenario concretely, this subsection considers a special case of our model with a single type of intermediary i. We that the regulator can freely set corrective taxes $\boldsymbol{\tau}_b^i$ on security issuance, but cannot directly control the intermediary's choice $\boldsymbol{k}^i$ of risky capital.

Proposition 4. *[Regulation with Unregulated Activities Within an Intermediary] If there is a single type $\mathcal{I} = \{i\}$ of intermediary, and the planner is constrained to set $\boldsymbol{\tau}_k^i = 0$, then the optimal tax $\boldsymbol{\tau}_b^i$ on security issuance satisfies*

$$\boldsymbol{\omega}_b^i = \boldsymbol{\tau}_b^i - \boldsymbol{\delta}_b^i = \boldsymbol{\delta}_k^i \cdot \frac{d\boldsymbol{k}^i}{d\boldsymbol{\tau}_b^i} \tag{26}$$

The left-hand side of Equation (26) is the wedge between the tax and marginal distortion on the regulated activity (security issuance). Once again, this wedge would be set to zero in a first-best world where the all activities are regulated. The right-hand side shows that the optimal wedge accounts for the distortion caused by the unregulated activity (capital investment) and its response to regulation.

For example, Dávila and Walther (2020*b*) consider a canonical model in which intermediaries conduct levered, risky investments. In this environment, risky capital investment by intermediaries increases the likelihood of bailouts, so that $\boldsymbol{\delta}_k^i > 0$, while stricter regulation of debt issuance increases the effective cost of capital investment, so that $\frac{d\boldsymbol{k}^i}{d\boldsymbol{\tau}_b^i} < 0$. In this scenario, Equation (26) implies that the optimal wedge on debt issuance is positive, implying $\boldsymbol{\tau}_b^i > \boldsymbol{\delta}_b^i$. Therefore, unregulated activities within intermediaries commonly imply taxes that are higher than the Pigou principle would suggest. The planner optimally over-corrects the activities she can control, so as to dampen externalities arising from complementary activities. Therefore, our analysis reveals that the implications of unregulated activity *within* intermediaries are different from those of leakage *across* intermediaries that we considered in the previous subsection.

3.5 The value of regulating the unregulated

We further use our analysis to evaluate recent policy reforms that enable governments to extend regulation to unregulated intermediaries. For example, the Dodd-Frank Act has extended regulation in the US to a wider set of intermediaries (Gorton et al., 2010), and the Basel III Accords impose liquidity transformation limits on some money market mutual funds (Adrian and Ashcraft, 2016). Formally, we can assess the value of such policies by returning to the scenario in which the planner can freely impose taxes only on one segment of intermediaries $i \in \mathcal{I}^\star \subset \mathcal{I}$. We study the second-best problem in which the planner is constrained to set $\tau_b^i \leq \bar{\tau}_b^i$ and $\tau_k^i \leq \bar{\tau}_k^i$ for all intermediaries $i \notin \mathcal{I}^\star$ in the unregulated segment.[7] The shadow value of relaxing these constraints measures of the marginal value of regulating the unregulated.

Proposition 5. *[Regulating the Unregulated] If the planner is constrained $\tau_b^i \leq \bar{\tau}_b^i$ and $\tau_k^i \leq \bar{\tau}_k^i$ for $i \notin \mathcal{I}^\star \subset \mathcal{I}$, then the marginal value of relaxing regulatory constraints for security issuance by intermediary $i \notin \mathcal{I}^\star$ is*

$$\sum_{j\in\mathcal{I}} \left(\boldsymbol{\omega}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} + \boldsymbol{\omega}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i} \right)$$

[7] The analysis in the previous subsection focused on the special case where taxes in the unregulated sector were constrained to zero.

The expression in Proposition 5 shows that the marginal value of regulating $i \notin \mathcal{I}_0$ depends not only on the distortions created by i, but on the substitution effects among all other intermediaries to this change in policy. For instance, consider a model with two intermediaries and fixed capital $k^i \equiv 1$. Consider a situation where $i = 1$ is initially unregulated, and both intermediaries are initially underregulated, in the sense that $\boldsymbol{\omega}_b^i \leq 0$. The direct welfare benefit of to imposing a small tax on intermediary 1 is $\boldsymbol{\omega}_b^1 \cdot \frac{d\boldsymbol{b}^1}{d\boldsymbol{\tau}_b^i}$, but the total welfare benefit is $\boldsymbol{\omega}_b^1 \cdot \frac{d\boldsymbol{b}^1}{d\boldsymbol{\tau}_b^1} + \boldsymbol{\omega}_b^2 \cdot \frac{d\boldsymbol{b}^2}{d\boldsymbol{\tau}_b^1}$. If securities are substitutes across intermediaries, we have $\frac{d\boldsymbol{b}^2}{d\boldsymbol{\tau}_b^1} \geq 0$, and the total welfare benefit is smaller than the direct one. Intuitively, leakage works in both directions, and also reduces the attractiveness of regulating previously unregulated entities.

3.6 Marginal value of one-size-fits-all regulation

As a final example of a second-best policy, we consider the value of policies that impose the same corrective taxes on all intermediaries. This example revisits the analysis of Diamond (1973) in the context of financial intermediation. In practical terms, this analysis sheds light on recent proposals that financial policy should be imposed at the level of activities (e.g., restrictions on short-term debt issuance by any intermediary), as opposed to institutions (e.g., regulation of commercial banks). For example, the FSOC (Financial Stability Oversight Council) has the power to require regulation of specific activities as well as specific entities, but has so far restricted itself to an entity-based approach (Kress, 2017). Our model can shed light on the statistics that determine the value of activity-based regulation.

Formally, assume that the planner can impose taxes on all intermediaries' activities, but is constrained to set $\boldsymbol{\tau}_b^i = \bar{\boldsymbol{\tau}}_b$ and $\boldsymbol{\tau}_k^i = \bar{\boldsymbol{\tau}}_k$ for all $i \in \mathcal{I}$. We show that the same weighted-average formula as in Proposition 1 applies in this scenario.

Proposition 6. *[One-size-fits-all Regulation/Diamond 1973 Revisited] If the planner is constrained to set $\boldsymbol{\tau}_b^i = \bar{\boldsymbol{\tau}}_b$ and $\boldsymbol{\tau}_k^i = \bar{\boldsymbol{\tau}}_k$ for all $i \in \mathcal{I}$, then any optimal policy satisfies:*

$$\frac{dW}{d\bar{\boldsymbol{\tau}}_b} = \sum_{j \in \mathcal{I}} \left(\boldsymbol{\omega}_b^j \cdot \frac{d\boldsymbol{b}^j}{d\bar{\boldsymbol{\tau}}_b} + \boldsymbol{\omega}_k^j \cdot \frac{d\boldsymbol{k}^j}{d\bar{\boldsymbol{\tau}}_k} \right) \tag{27}$$

It is worth highlighting here that imposing the same regulation to all intermediaries is only restrictive when the distortion associated with the different activities are asymmetric across intermediaries. That is, if all intermediaries are symmetric, it is possible to achieve the first-best outcome, an idea that we exploit in our application next.

4 Application: shadow banking

Shadow banking typically describes the financial activities occurring outside of the regulated financial sector. In this section, we study an application of our general framework to the problem of shadow banking by considering an environment with types of intermediary in which only one type can be regulated. In particular, we provide explicit illustrations of our main results introduced in Section 3, with an emphasis on connecting leakage elasticities to model primitives.

Environment As in the general model, there are two dates, 0 and 1 and a continuum of states $s \in [\underline{s}, \overline{s}]$ with cdf $F(s)$. We consider an environment with two (types of) price-taking intermediaries, indexed by $i = \{1, 2\}$, a unit measure of creditors, indexed by C, and a government.

Intermediaries are risk-neutral, with preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s). \tag{28}$$

Each intermediary has access to a linear technology. An investment at date 0 of $k^i \geq 0$ units of capital yields $sv^i k^i$ dollars in state s at date 1. Investment is subject to a convex adjustment cost $\Upsilon^i(k^i) = \frac{a^i}{2}(k^i)^2$. Intermediaries finance their investment by issuing bonds with face value b^i per unit of investment (i.e., the total stock of debt issued is $b^i k^i$, and an intermediary's leverage ratio is simply b^i). Any remaining financing is obtained with an equity contribution from the intermediary's endowment. Formally, the date 0 budget constraint of an intermediary is given by

$$c_0^i + k^i + \Upsilon^i\left(k^i\right) = n_0^i + Q^i\left(b^i, r\right) k^i - \tau_b^i b^i k^i - \tau_k^i k^i, \tag{29}$$

where the financing schedule $Q^i(b^i, r)$ is determined by creditors as described below. Since b^i is expressed as face value per unit of investment, intermediaries face financing taxes given by $\tau_b^i b^i k^i$. Investment taxes are simply given by $\tau_k^i k^i$.

At date 1, after the state s is realized, intermediaries decide whether to default. After default, creditors seize all of the intermediaries' resources and receive a fraction ϕ of the investment returns. The remaining fraction $(1-\phi)$ measures the deadweight loss associated with default.

We assume that creditors are offer financing-schedules risk-neutrally, using a discount factor $\beta^C(r) = \frac{1}{1+r}$, where r is determined in equilibrium by clearing the credit market. Formally, to simplify the exposition, we take as primitive a supply of credit $Q_S(r)$, where $Q_S(r) \geq 0$ and $Q_S'(r) > 0$. In particular, we assume that $Q_S(r^\star) = s_0 + s_1 r$. In order to guarantee that intermediaries borrow from creditors, we assume that $0 < \beta^I \leq \beta^C(r) \leq 1$, $\forall r$, so that intermediaries are more impatient than creditors.[8]

Finally, regarding the government, we assume throughout that the bailout policy is given by

$$t^i\left(b^i, k^i, s\right) = \left(\alpha_0 - \alpha_s s + \alpha_b b^i\right) k^i,$$

where α_0, α_s, and α_b are non-negative scalars. Below, we will consider different scenarios regarding the form of the regulatory policy, which in this model is fully characterized by $\{\boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i\} = \{\tau_b^1, \tau_b^2, \tau_k^1, \tau_k^2\}$. Since we focus on aggregate welfare, which we assume to be computed under a utilitarian criterion, our results are valid regardless of how tax revenues are rebated.

The equilibrium definition in this environment mirrors the one considered in Section 2. An *equilibrium* with corrective taxes $\{\boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i, T_0^i\}_{i\in\mathcal{I}}$ and ex-post bailout transfers $\left\{\{t^i(b^i, k^i, s)\}_{i\in\mathcal{I}}, T_1(s)\right\}$ consists of consumption bundles $\{c_0^i, c_1^i(s)\}_{i\in\mathcal{I}}$ and $\left\{c_0^C, c_1^C(s)\right\}$, intermediaries' financing and investment decisions $\{b^i, k^i\}_{i\in\mathcal{I}}$, financing schedules $Q^i(b^i, r)$, an equilibrium interest rate $r^\star$, such that i) intermediaries maximize utility subject to budget constraints taking the interest rate as given but internalizing how their actions affect the repayment of creditors, ii) creditors offer a financing schedule risk-neutrally for a given interest rate r, iii) the government's budget constraints are satisfied, and iv) the financing market clears, that is,

$$\sum_i Q^i\left(b^{i\star}\left(r^\star, \tau_b^i, \tau_k^i\right); r^\star\right) k^i = Q_S(r^\star). \tag{30}$$

It is worth highlighting that the only connection between intermediaries 1 and 2 in this model occurs through changes in the interest rate. This is the simplest environment that allows us to illustrate our general insights regarding the form of the optimal corrective policies.

Equilibrium characterization As shown in the Appendix, the objective function of intermediaries is given by

$$J^i\left(b^i, k^i; r, \tau_b^i, \tau_k^i\right) = \left[E^i\left(b^i\right) + Q^i\left(b^i, r\right) - 1 - \tau_b^i b^i - \tau_k^i\right] k^i - \Upsilon^i\left(k^i\right),$$

[8] As we show in the Appendix, this formulation with a given credit supply and risk-neutral can be mapped to an environment in which creditors have preferences of the form $u\left(c_0^C\right) + \beta^C \mathbb{E}\left[u\left(c_1^C(s)\right)\right]$. These preferences impply that creditors do not require to be compensated by holding risk, but are unwilling to fully substitute intertemporally

where $E^i(b^i)$ and $Q^i(b^i, r)$ respectively denote the market value of equity and debt per unit of capital, given by

$$E^i\left(b^i\right) = \beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} \left(sv^i + t^i\left(b^i, s\right) - b^i\right) dF(s)$$

$$Q^i\left(b^i, r\right) = \beta(r)\left(\int_{s^{i\star}(b^i)}^{\overline{s}} b^i dF(s) + \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} \left(\phi^i sv^i + t^i\left(b^i, s\right)\right) dF(s)\right),$$

and where the optimal decision of intermediaries imply that $s^{i\star}(b^i) = \left(\frac{1-\alpha_b}{v^i-\alpha_s}\right) b^i - \frac{1}{v^i-\alpha_s}\alpha_0$. The solution to this problem yields demand functions for both credit and investment, which we denote by $b^{i\star}(r, \tau_b^i, \tau_k^i)$ and $k^{i\star}(r, \tau_b^i, \tau_k^i)$. As we explain in the Appendix, the problem of intermediaries can be solved sequentially, since the choice of leverage by intermediaries b^i is independent of the scale of investment k^i.

We denote the indirect utility of intermediaries, given an interest rate r, by[9]

$$V^i\left(r, \tau_b^i, \tau_k^i, T_0^i\right) = J^i\left(b^{i\star}\left(r, \tau_b^i, \tau_k^i\right), k^{i\star}\left(r, \tau_b^i, \tau_k^i\right); r, \tau_b^i, \tau_k^i\right) + T_0^i,$$

where the revenue raised is given by

$$T_0^i\left(b^{i\star}, k^{i\star}\right) = \tau_b^i b^{i\star} k^{i\star} - \tau_k^i k^{i\star}.$$

The expected bailout per unit of capital is given by

$$\mathcal{B}^i = \beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} t^i\left(b^i, s\right) dF(s) + \beta(r) \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} t^i\left(b^i, s\right) dF(s),$$

and the total amount of bailout funds raised is.

$$-(1+\kappa)\sum_i \mathcal{B}^i k^{i\star}.$$

The market clearing conditions for r equals the demand of credit to the supply of credit at date 0. Formally, the equilibrium interest rate $r^\star(\boldsymbol{\tau}_b^i, \boldsymbol{\tau}_k^i)$ is given by the solution to

$$\sum_i Q^i\left(b^{i\star}\left(r^\star, \tau_b^i, \tau_k^i\right); r^\star\right) k^{i\star}\left(r^\star, \tau_b^i, \tau_k^i\right) = Q_S(r^\star).$$

Parametrization and results Going forward, to more clearly illustrate the impact of not being able to regulate the second type of intermediaries, we assume i) that $k^1 = k^2 = 1$,

[9] Because agents are risk-neutral and because we consider a utilitarian welfare criterion, λ_0^i as defined in Section 3, equals 1.

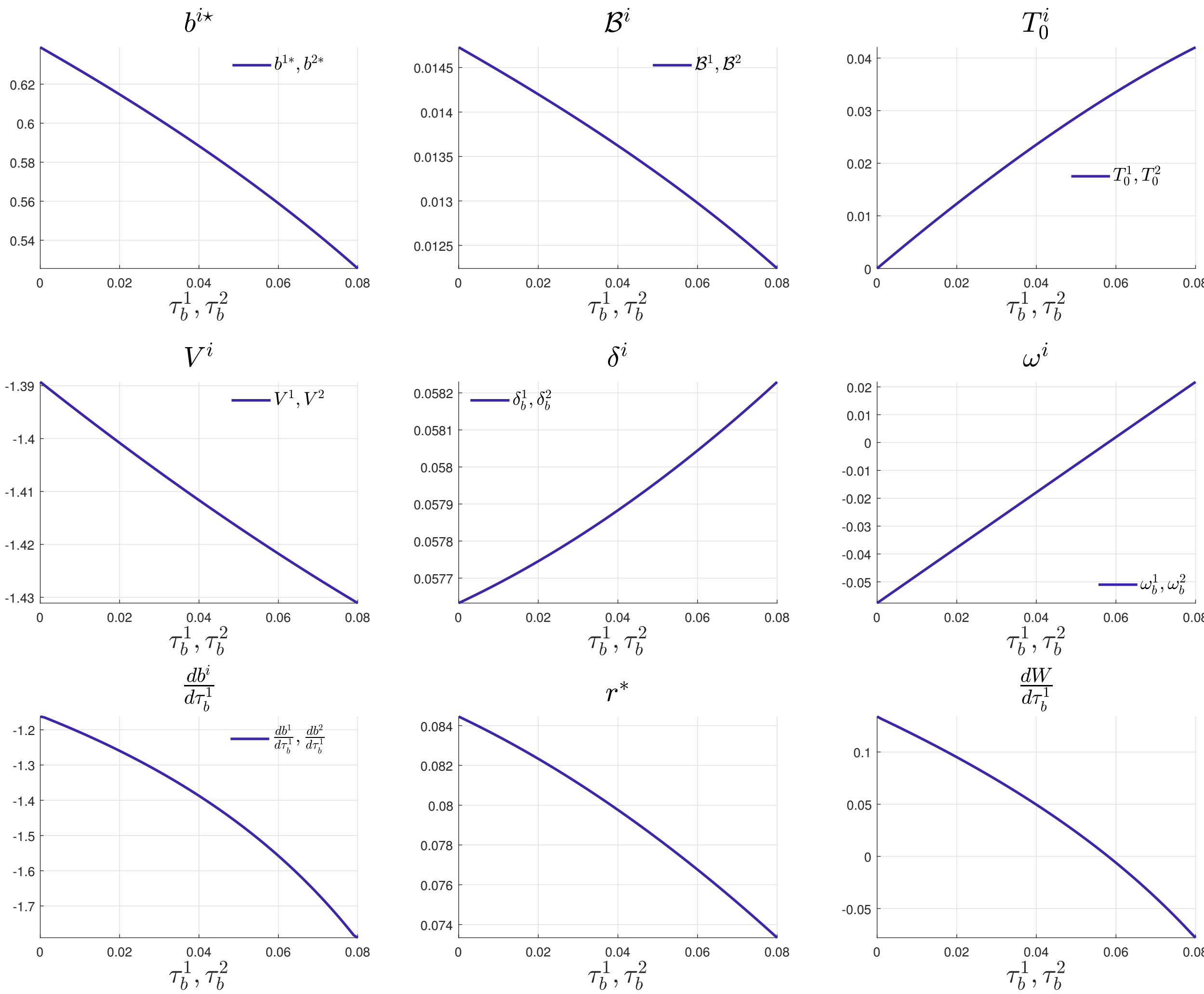

Figure 1: First-best

Note: Figure 1 shows the outcome of the model developed in Section 4, for different values of τ_b^1, which equals τ_b^2. The top plots in Figure 1 show the equilibrium level of leverage $b^{i\star}$, the size of the bailout $\mathcal{B}^i$, and the equilibrium tax revenue T_0^i. The middle plots in Figure 1 show the indirect utility of intermediaries V^i, the magnitude of the distortion δ^i, and the magnitude of the Pigouvian wedge ω^i for different values of $\tau_b^1 = \tau_b^2$. The bottom plots in Figure 1 show the leverage leakage elasticity $\frac{db^i}{d\tau_b^i}$, the equilibrium interest rate r^*, and the marginal welfare effects of a tax change $\frac{dW}{d\tau_A^i}$ for different values of $\tau_b^1 = \tau_b^2$. The parameters used in all plots are $\beta^i = 0.8$, $\phi = 0$, $a^i = 1.5$, $\alpha_0 = \alpha_s = 0$, $\alpha_b = 0.025$, $v^i = 1$, $s_0 = 0.8$ and $s_1 = 1$. For reference, the optimal first-best leverage tax is 0.0581, while the the optimal tax when the second type of intermediaries cannot be regulated is 0.0554.

ii) that $\alpha_0 = \alpha_s = 0$, iii) that intermediaries are symmetric, but for the possibility that intermediary 2 cannot be taxed. Given our assumptions, the expected bailout term simplifies to $\mathcal{B}^i = \beta(r)\,\alpha_b b^i$, which implies that the Pigouvian wedge for leverage is given by

$$\omega_b^i = \tau_b^i - \delta_b^i,$$

where and the distortion corresponds to

$$\delta_b^i = (1+\kappa)\frac{\partial \mathcal{B}^i}{\partial b^i} k^{i\star} = (1+\kappa)\,\beta(r)\,\alpha_b.$$

In this case, it follows directly from Proposition 1 that $\frac{dW}{d\tau_b^j} = \sum_i \left(\tau_b^i - \delta_b^i\right) \frac{db^i}{d\tau_b^j} k^i$.

We use the following parameters. We assume that intermediaries have a discount factor $\beta^i = 0.8$, that their recovery rate is $\phi = 0$, $a^i = 1.5$, $\alpha_b = 0.025$, $v^i = 1$, and that the supply of credit is determined by $s_0 = 0.8$ and $s_1 = 1$.

We graphically illustrate two different scenarios. First, in Figure 1, we show different equilibrium objects when varying the tax rate on leverage for both types of intermediaries. Given the symmetry assumption, it is easy in this case to find the first-best solution to the problem, which is achieved when $\frac{dW}{d\tau_b} = 0$. The top plots in Figure 1 show the equilibrium level of leverage $b^{i\star}$, the size of the bailout $\mathcal{B}^i$, and the equilibrium tax revenue T_0^i. The middle plots in Figure 1 show the indirect utility of intermediaries V^i, the magnitude of the distortion δ^i, and the magnitude of the Pigouvian wedge ω^i for different values of $\tau_b^1 = \tau_b^2$. The bottom plots in Figure 1 show the leverage leakage elasticity $\frac{db^i}{d\tau_b^i}$, the equilibrium interest rate r^*, and the marginal welfare effects of a tax change $\frac{dW}{d\tau_A^i}$ for different values of $\tau_b^1 = \tau_b^2$. As one would expect, an increase in the tax on leverage reduces equilibrium borrowing and equilibrium interest rates. The reduction in interest rates, by discounting less the future, increases the magnitude of the bailout distortion at date 1, increasing also the size of the bailout. As shown in Proposition 2, it is sufficiently to consider the behavior of ω_b^i to find the optimal leverage regulation in this case.

The second scenario maps to our analysis in Section 3.3. In Figure 2, we show different equilibrium objects when varying the tax rate on leverage for intermediary 1, while setting the tax for intermediary 2 to zero. This is case in which a set of intermediaries is unregulated. In this case, an increase in the tax faced by intermediary 1 naturally lowers the amount of equilibrium leverage by intermediary 1, but it also increases the amount of leverage taken by intermediary, so $\frac{db^2}{d\tau_b^1} > 0$, which corresponds to the case in which the funding choices of intermediaries are gross substitutes. This intuitively occurs here because interest rates drop, due to the overall declined in borrowing demand. According to our results in Section 3, the fact that $\omega_b^2 < 0$ and $\frac{db^2}{d\tau_b^1} > 0$, implies that the optimal corrective tax with imperfect

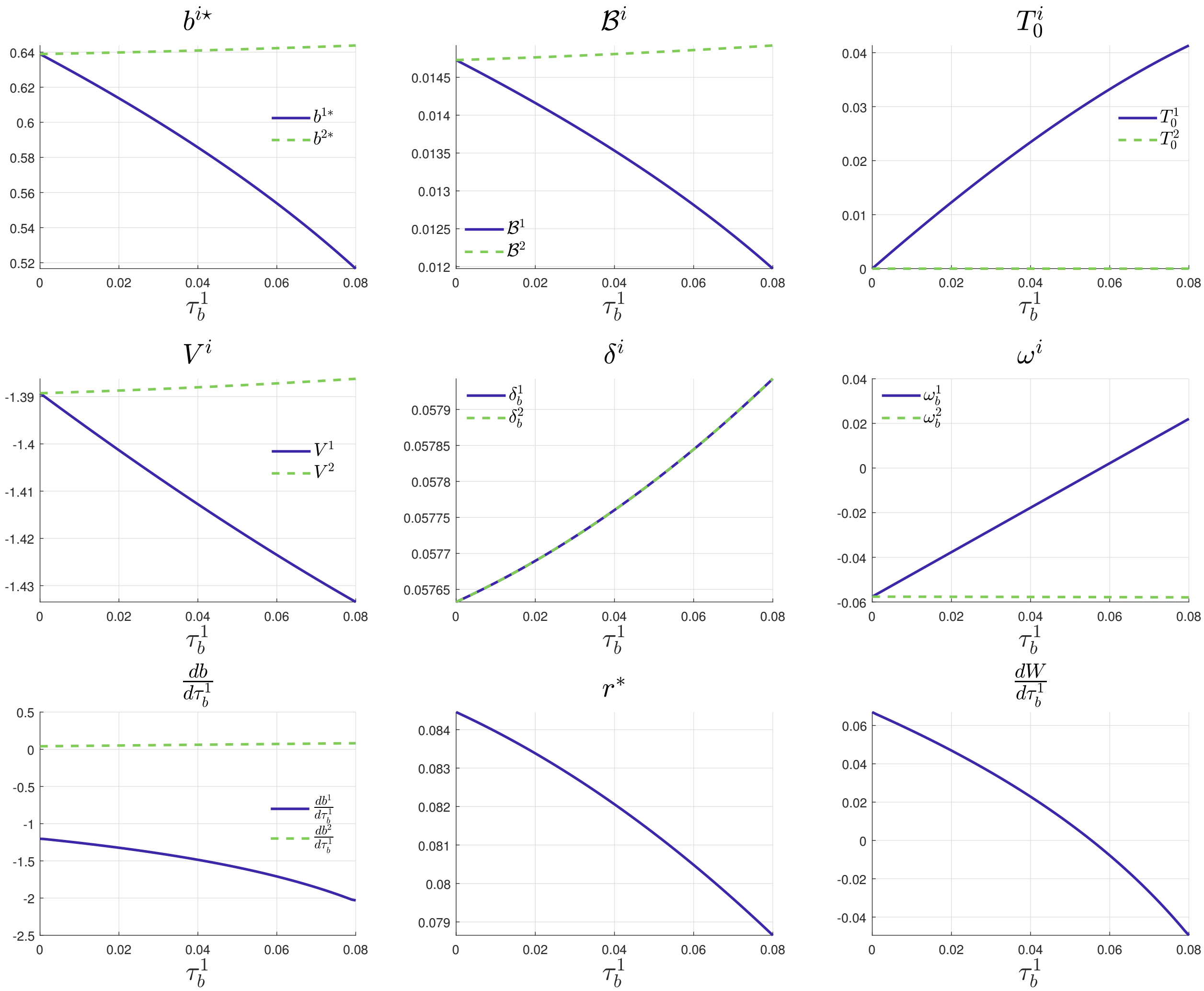

Figure 2: Optimal regulation with unregulated intermediaries

Note: Figure 2 shows the outcome of the model developed in Section 4, for different values of τ_b^1, when $\tau_b^2 = 0$. The top plots in Figure 2 show the equilibrium level of leverage $b^{i\star}$, the size of the bailout $\mathcal{B}^i$, and the equilibrium tax revenue T_0^i. The middle plots in Figure 2 show the indirect utility of intermediaries V^i, the magnitude of the distortion δ^i, and the magnitude of the Pigouvian wedge ω^i for different values of $\tau_b^1 = \tau_b^2$. The bottom plots in Figure 2 show the leverage leakage elasticity $\frac{db^i}{d\tau_b^i}$, the equilibrium interest rate r^*, and the marginal welfare effects of a tax change $\frac{dW}{d\tau_A^i}$ for different values of $\tau_b^1 = \tau_b^2$. The parameters used in all plots are $\beta^i = 0.8$, $\phi = 0$, $a^i = 1.5$, $\alpha_0 = \alpha_s = 0$, $\alpha_b = 0.025$, $v^i = 1$, $s_0 = 0.8$ and $s_1 = 1$. For reference, the optimal first-best leverage tax is 0.0581, while the the optimal tax when the second type of intermediaries cannot be regulated is 0.0554.

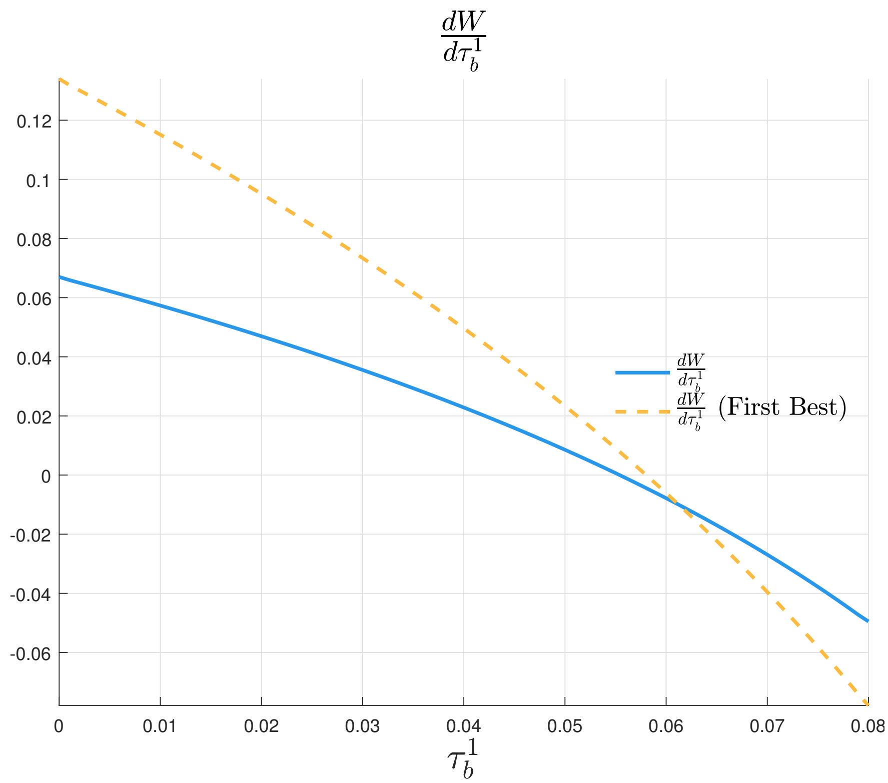

Figure 3: Marginal welfare effect comparison

Note: Figure 3 compares the marginal welfare effects of a tax change in the first-best scenario, in which τ_B^2 is also changing, with those in the scenario in which $\tau_B^2 = 0$. The parameters used in this plot are $\beta^i = 0.8$, $\phi = 0$, $a^i = 1.5$, $\alpha_0 = \alpha_s = 0$, $\alpha_b = 0.025$, $v^i = 1$, $s_0 = 0.8$ and $s_1 = 1$. For reference, the optimal first-best tax is 0.0581, while the the optimal tax when the second type of intermediaries cannot be regulated is 0.0554.

instruments is lower than the first-best tax. For reference, the optimal first-best leverage tax is 0.0581, while the the optimal tax when the second type of intermediaries cannot be regulated is 0.0554. Finally, Figure 3 compares the marginal welfare effects of a tax change in the first case consider, which illustrates the first-best solution, and the second one, which explains the optimal regulation when some intermediaries are unregulated. It illustrates our results in Propositions 1 and 3.

5 Direct measurement of leakage elasticities

A central implication of our results is that, in order to assess financial policy reforms with imperfect instruments, we need to measure distortions and leakage elasticities. In this section, we connect this reasoning to existing empirical evidence, which provides direct measurement of the relevant statistics. We revisit empirical work relating to three examples that we considered in Section 3: Optimal regulation with leakage across intermediaries (i.e., unregulated institutions), leakage within intermediaries (i.e., unregulated activities), and the effect of one-size-fits-all policies that effect all intermediaries.

5.1 Elasticities from regulated bank policy to other intermediaries

In Section 3.3 and Section 4, we shows that the needed to calibrate optimal policy with unregulated intermediaries are the relevant leakage elasticities, and the marginal distortions (or externalities) imposed by unregulated intermediaries.

The responsiveness of unregulated financial intermediaries to policy is the subject of a growing literature. To put this literature into the context of our model, we revisit the case presented in Section 4, in which there are regulated and unregulated segments of intermediaries. Optimal regulation depends on both the sensitivity of security issuance ($\frac{d\boldsymbol{b}^j}{d\boldsymbol{b}^i}$) and capital investment ($\frac{d\boldsymbol{k}^j}{d\boldsymbol{b}^i}$) by unregulated banks to restrictions on regulated capital ratios $\boldsymbol{b}^i$.

Buchak et al. (2018*b*) analyze US residential mortgages, where the share of loans originated by "shadow banks" has recently risen to about 50%.[10] They estimate the percentage response of shadow bank lending (k^j) to regulated capital ratios as $\frac{\mathrm{dlog}k^j}{db^i} \simeq 3.412$.[11] Moreover, the response of the market share of shadow banks is similar at $\frac{\mathrm{dlog}\left(k^j/\left(k^i+k^j\right)\right)}{db^i} \simeq 3.358$, which suggests that there is roughly one-for-one substitution between regulated and shadow banks with $\frac{dk^i}{db^i} \simeq -\frac{dk^j}{db^i}$. Buchak et al. (2018*a*) find similar responses with $\frac{\mathrm{dlog}\left(k^j/\left(k^i+k^j\right)\right)}{db^i} \simeq 3.508$ using a structural model of bank / shadow bank competition.[12]

Irani et al. (2020) study syndicated term loans in the US, where the market share non-bank intermediaries has risen to around 70%. They estimate the response of the market share of non-banks ($\frac{k^j}{k^i+k^j}$) to the capital ratios of regulated banks who, at the time of sampling, anticipated a substantial increase in required capital ratios under Basel III. One can loosely interpret a one-unit increase in current capital as a one-unit increase in the bank's effectively leverage limit b^i on new loans, so that the estimates imply that $\frac{dk^j/\left(k^i+k^j\right)}{db^i} \simeq 1.547$. All of the above estimates suggest gross substitutability between leverage b^i in the regulated sector and unregulated investments k^j. This discussion also highlights what is currently missing from the empirical literature. We are not aware of estimates of $\frac{db^j}{db^i}$, that is, the response of unregulated *leverage* b^j to regulated leverage b^i. This is another important ingredient that would need to be measured to fully determine optimal policy.

[10] In Buchak et al. (2018*b*), the term shadow bank refers to any mortgage originator that is not a regulated bank.

[11] This estimate is taken from Table 8, column (5) in Buchak et al. (2018*b*).

[12] This estimate is extracted from Table 9 in Buchak et al. (2018*a*) as follows: In a counterfactual scenario where regulated capital ratios rise from $b^i = 6\%$ to 9%. The percentage response in shadow banks' market share is $\left(\frac{42}{38}-1\right) = 10.526\%$, which implies $\frac{\mathrm{dlog}\left(k^j/\left(k^i+k^j\right)\right)}{db^i} = 3.508$ when b^i is measured in percentage points.

The second set of statistics needed to calibrate optimal policy consists of the marginal distortions δ_b^i and δ_k^i that are caused by both regulated and unregulated intermediaries (see Equation (20) for their definition). Following the crisis of 2008, a popular approach to measurement has been to examine co-movement between market data on individual institutions' distress, and data on distress of the financial system as a whole (e.g., Acharya, Engle and Richardson, 2012; Adrian and Brunnermeier, 2016). This is consistent with distortions in our model: Distortions are proportional to the marginal cost of fiscal intervention $\mu(s)$, which is in turn increasing in the system-wide measure of bailouts $T_1(s)$.

One benefit of these recent methods is that they use market data to define distress, and are therefore observable not only for regulated institutions, but also for large, traded non-bank intermediaries such as large insurance companies and families of mutual funds. These institutions tend to have marginal measured distortions that are comparable, in sign and magnitude, to those of large banks.[13]

In summary, existing measurements in developed economies suggest that unregulated intermediaries (i) act as substitutes for the regulated sector, and (ii) impose positive marginal externalities. Combined with Proposition 3, this evidence calls for "underregulation" in the regulated sector, which stops short of aligning private and social marginal costs of leverage. However, as discussed in Section 3.3 , underregulation in the sense of wedges between social and private marginal cost may still be consistent with higher overall levels of intervention (i.e., higher corrective taxes or, equivalently, tighter binding capital requirements) if the distortions caused by increased unregulated activity are large enough.

Finally, a recent case study of the Chinese financial system by Allen et al. (2019) suggests that a more nuanced approach may be needed in other institutional circumstances. In particular, Allen et al. (2019) focus on Chinese non-financial firms who borrow from banks and finance other firms' projects, thus effectively acting as unregulated intermediaries. This practice constitutes a large part of the Chinese shadow banking system. An intriguing result of their analysis is that unregulated lending activity *expands* when credit is tight in general. This is suggestive evidence that some forms of unregulated intermediation, particularly if undertaken by firms with strong balance sheets, can remove system-wide instability. In our model, this case corresponds to *negative* marginal distortions δ_b^i and δ_k^i for unregulated intermediaries, and therefore implies that optimal policy should encourage substitution into unregulated lending by imposing higher taxes on the regulated segment.

[13]See, for example: https://vlab.stern.nyu.edu/analysis/RISK.USFIN-MR.MES

5.2 Elasticities within intermediaries

In Section 3.4, we considered optimal regulation of an intermediary who conducts unregulated activities. One example, motivated by the analysis in Dávila and Walther (2020*b*), is where the regulator can impose restrictions or taxes on leverage, but cannot control risky capital investment. A number of recent empirical studies confirm that the leakage elasticity from leverage to risky investments is positive, in the sense that banks with lower capital ratios originate a larger volume of risky loans (see, for example, Jiménez et al. (2014), Dell'Ariccia, Laeven and Suarez (2017), Acharya et al. (2018) and references therein). As we argued above, this case has the opposite policy implication from the case of unregulated intermediaries. Indeed, if leverage and risky investments are complementary, optimal leverage policy should be stricter than the Pigou principle suggests.

5.3 Elasticities for one-size-fits-all regulation

Our analysis in Section 3.6, where we consider the effect of one-size-fits-all policy, can be used to shed light on recent evidence regarding monetary policy. In particular, Xiao (2020*b*) shows that loan volumes respond to monetary in opposite direction for regulated and unregulated intermediaries in the US, with "shadow banks" substituting for regulated banks' lending when monetary policy becomes tighter in the sense of a higher Fed funds rate. The policy implications of this result can be viewed through he lens of Proposition 6. The proposition highlights that the optimal policy should be chosen based on the *aggregate* responsiveness, weighted by marginal distortions, of regulated and unregulated intermediaries. Xiao (2020*b*) uses an explicit structural model to show that this aggregate responsiveness is severely muted in the presence of unregulated intermediaries. Hence, in conjunction with our normative analysis, his results imply that the case for contractionary monetary policy is weakened in the presence of unregulated intermediaries.

6 Conclusion

This paper provides a systematic study of optimal financial regulation with imperfect instruments. We have shown that that leakage elasticities and Pigovian wedges are sufficient statistics to account for the marginal welfare impact of imperfect regulatory policies in a large class of environments. We have characterize the optimal regulatory policy with unregulated intermediaries, the optimal regulatory policy with regulated and unregulated activities within intermediaries, the value of regulating unregulated intermediaries, and the optimal one-size-fit-all regulation. We have leveraged our approach to reinterpret a number of results in the growing empirical literature on shadow banking. Even though we have chosen shadow banking to quantitatively illustrate our insight, there is scope to further apply the principles identified in this paper in other forms of financial regulation, for instance, when considering the regulation of capital markets or household financial activities.

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A Proofs and derivations: Section 2

The problem solved by intermediary i in Lagrangian form is

$$\max_{c_0^i, \left\{c_1^i(s)\right\}, \boldsymbol{b}^i, \boldsymbol{k}^i} \mathcal{L}^i,$$

where $\mathcal{L}^i$ is given by

$$\begin{aligned}
\mathcal{L}^i &= u_0^i\left(c_0^i\right) + \beta^i \int u_1^i\left(c_1^i\left(s\right)\right) dF\left(s\right) \\
&- \lambda_0^i\left(c_0^i - n_0^i - Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) + \boldsymbol{\tau}_b^i \cdot \boldsymbol{b}^i + \left(\boldsymbol{p}_k + \boldsymbol{\tau}_k^i\right) \cdot \boldsymbol{k}^i + \Upsilon^i\left(\boldsymbol{k}^i\right) - T_0^i\right) \\
&- \int \left[\lambda_1^i\left(s\right)\left(c_1^i\left(s\right) - n_1^i\left(s\right) - \max\left\{v^i\left(\boldsymbol{k}^i, s\right) + t^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \Omega^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right), 0\right\}\right)\right] dF\left(s\right),
\end{aligned}$$

where λ_0^i and $\lambda_1^i\left(s\right)$ denote the Lagrange multipliers that correspond to the intermediary budget constraints. The optimality conditions for intermediaries on $\boldsymbol{b}^i$ and $\boldsymbol{k}^i$ are

$$\begin{aligned}
\lambda_0^i\left(\frac{\partial Q^i}{\partial \boldsymbol{b}^i} - \boldsymbol{\tau}_b^i\right) - \int_{\mathcal{N}} \lambda_1^i\left(s\right)\left(-\frac{\partial t^i}{\partial \boldsymbol{b}^i}\left(s\right) + \frac{\partial \Omega^i}{\partial \boldsymbol{b}^i}\left(s\right)\right) dF\left(s\right) = 0 \\
\lambda_0^i\left(\frac{\partial Q^i}{\partial \boldsymbol{k}^i} - \left(\boldsymbol{p}_k + \boldsymbol{\tau}_k^i + \frac{d\Upsilon^i}{d\boldsymbol{k}^i}\right)\right) - \int_{\mathcal{N}} \lambda_1^i\left(s\right)\left(\frac{\partial v^i}{\partial \boldsymbol{k}^i}\left(s\right) - \frac{\partial t^i}{\partial \boldsymbol{k}^i}\left(s\right) + \frac{\partial \Omega^i}{\partial \boldsymbol{k}^i}\right) dF\left(s\right) = 0,
\end{aligned}$$

where the set $\mathcal{N}$ denotes the default region. These expressions correspond to Equations (13) and (14) in the text.

The problem solved by creditors in Lagrangian form is

$$\max_{c_0^C, \left\{c_1^C(s)\right\}, \left\{d_i^C\right\}} \mathcal{L}^C,$$

where $\mathcal{L}^C$ is given by

$$\begin{aligned}
\mathcal{L}^C &= u_0^C\left(c_0^C\right) + \beta^C \int u_1^C\left(c_1^C\left(s\right)\right) dF\left(s\right) \\
&- \lambda_0^C\left(c_0^C - n_0^C + \sum_{i \in \mathcal{I}} d_i^C Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) - \Pi_k\left(\boldsymbol{p}_k\right)\right) \\
&- \int \lambda_1^C\left(s\right)\left(n_1^C\left(s\right) + \sum_{i \in \mathcal{I}} d_i^C \tilde{\Omega}^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) - \kappa\left(T_1\left(s\right), s\right)\right) dF\left(s\right),
\end{aligned}$$

where λ_0^C and $\lambda_1^C\left(s\right)$ denote the Lagrange multipliers that correspond to the creditors budget constraints. The optimality conditions for creditors on $\left\{d_i^C\right\}$ are

$$\lambda_0^C Q^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i\right) - \int \lambda_1^C(s)\,\tilde{\Omega}^i\left(\boldsymbol{b}^i, \boldsymbol{k}^i, s\right) dF(s) = 0.$$

This expression corresponds to Equation (12) in the text.

B Proofs and derivations: Section 3

Proof of Lemma 1

Proof. For financing schedules, note that the maximization problem of intermediaries $j \neq i$ does not depend on prices Q^i faced by intermediary i, so we have $\frac{\partial V^j}{\partial Q^i} = 0$. Therefore, for intermediary i, we have that

$$\frac{\partial W}{\partial Q^i} = \frac{\frac{\partial \tilde{V}^C}{\partial Q^i}}{\lambda_0^C} + \frac{\frac{\partial \tilde{V}^i}{\partial Q^i}}{\lambda_0^i}.$$

For creditors, Roy's identity implies that

$$\frac{\partial \tilde{V}^C}{\partial Q^i} = -\lambda_0^C d_i^C = -\lambda_0^C$$

where the second equality follows from the market clearing condition $d_i^C = 1$. For intermediary i, Roy's identity implies that

$$\frac{\partial \tilde{V}^i}{\partial Q^i} = -\lambda_0^i,$$

which establishes Equation (15).

For capital prices, note that for creditors

$$\frac{\partial \tilde{V}^C}{\partial \Pi_k} = \lambda_0^C,$$

and that Hotelling's lemma implies that

$$\frac{\partial \Pi_k}{\partial \boldsymbol{p}_k} = \boldsymbol{k}^S.$$

For intermediaries, Roy's identity implies that

$$\frac{\partial \tilde{V}^i}{\partial \boldsymbol{p}_k} = -\lambda_0^i \boldsymbol{k}^i.$$

Combining, we obtain

$$\begin{aligned}\frac{\partial W}{\partial \boldsymbol{p}_k} &= \frac{\frac{\partial \tilde{V}^C}{\partial \Pi_k}\frac{\partial \Pi_k}{\partial \boldsymbol{p}_k}}{\lambda_0^C} + \sum_{i \in \mathcal{I}} \frac{\frac{\partial \tilde{V}^i}{\partial \boldsymbol{p}_k}}{\lambda_0^i} \\ &= \boldsymbol{k}^S - \sum_{i \in \mathcal{I}} \boldsymbol{k}^i = 0\end{aligned}$$

where the last equality follows from capital market clearing. □

Proof of Proposition 1

Proof. The effect of raising $\boldsymbol{\tau}_b^i$ on the indirect utility of creditors, ignoring pecuniary effects, is

$$\begin{aligned}\frac{d\tilde{V}^C}{d\boldsymbol{\tau}_b^i} &= \int \frac{\partial V^C}{\partial T_1(s)} \frac{dT_1(s)}{d\boldsymbol{\tau}_b^i} dF(s) \\ &= -\sum_{s \in \mathcal{S}} \kappa'(T_1(s))\, \lambda_1^C(s) \frac{dT_1(s)}{d\boldsymbol{\tau}_b^i} \\ &= -\lambda_0^C \sum_{s \in \mathcal{S}} \kappa'(T_1(s)) \frac{\lambda_1^C(s)}{\lambda_0^C} \sum_{j \in \mathcal{I}} \left(\frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{b}^j} \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} + \frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{k}^j} \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i} \right) \\ &= -\lambda_0^C \sum_{j \in \mathcal{I}} \left(\mathbb{E}\left[\gamma(s) \frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{b}^j} \right] \frac{d\boldsymbol{b}^j}{d\boldsymbol{\tau}_b^i} + \mathbb{E}\left[\gamma(s) \frac{\partial t^j\left(\boldsymbol{b}^j, \boldsymbol{k}^j, s\right)}{\partial \boldsymbol{k}^j} \right] \frac{d\boldsymbol{k}^j}{d\boldsymbol{\tau}_b^i} \right) \qquad (31)\end{aligned}$$

where the third line follows from differentiating the government's budget constraint (9), and the final equality by substituting the definition of $\mu(s)$ from (17). The effect of raising $\boldsymbol{\tau}_b^i$ on the indirect utility of an intermediary $j \neq i$ consists only of pecuniary effects, which we can ignore, since $V^j\left(Q^j, \boldsymbol{p}_k, \boldsymbol{\tau}_b^j, \boldsymbol{\tau}_k^j, T_0^j\right)$ does not depend directly on $\boldsymbol{\tau}_b^i$.

To calculate the effect of raising $\boldsymbol{\tau}_b^i$ on the indirect utility of intermediary i, ignoring pecuniary effects, is

$$\frac{dV^i}{d\boldsymbol{\tau}_b^i} = \frac{\partial V^i}{\partial \boldsymbol{\tau}_b^i} + \frac{\partial V^i}{\partial T_0^i} \frac{dT_0^i}{d\boldsymbol{\tau}_b^i}$$

Notice that $\frac{\partial V^i}{\partial \tau_b^i} = -\lambda_0^i \boldsymbol{b}^i$ and $\frac{\partial V^i}{\partial T_0^i} = \lambda_0^i$. Moreover, totally differentiating the government's budget constraint (8), we have

$$\begin{aligned}\frac{dT_0^i}{d\boldsymbol{\tau}_b^i} &= \frac{d}{d\boldsymbol{\tau}_b^i}\left[\boldsymbol{\tau}_b^i \boldsymbol{b}^i + \boldsymbol{\tau}_k^i \boldsymbol{k}^i\right] \\ &= \boldsymbol{b}^i + \boldsymbol{\tau}_b^i \frac{d\boldsymbol{b}^i}{d\boldsymbol{\tau}_b^i} + \boldsymbol{\tau}_k^i \frac{d\boldsymbol{k}^i}{d\boldsymbol{\tau}_b^i}\end{aligned}$$

Hence,

$$\frac{dV^i}{d\boldsymbol{\tau}_b^i} = \lambda_0^i \left(\boldsymbol{\tau}_b^i \frac{d\boldsymbol{b}^i}{d\boldsymbol{\tau}_b^i} + \boldsymbol{\tau}_k^i \frac{d\boldsymbol{k}^i}{d\boldsymbol{\tau}_b^i} \right) \tag{32}$$

Combining (31) and (32), we get Equation (18). Equation (19) follows from a parallel argument. □

C Proofs and derivations: Section 4

Given our formulation of bailouts $t^i\left(b^i, s\right) = \alpha_0 - \alpha_s s + \alpha_b b^i$, we express the default threshold $s^{i\star}\left(b^i\right)$ as follows

$$s^{i\star}\left(b^i\right) = \left(\frac{1-\alpha_b}{v^i - \alpha_s}\right) b^i - \frac{1}{v^i - \alpha_s}\alpha_0.$$

Note that $\frac{\partial t^i}{\partial b}\left(b^i, s\right) = \alpha_b$ and $\frac{\partial t^i}{\partial s}\left(b^i, s\right) = -\alpha_s$.

Formally, the objective function of intermediaries can be written as

$$\begin{aligned} J^i\left(b^i, k^i; r, \tau_b^i, \tau_k^i\right) &= \left[M^i\left(b^i\right) - 1\right] k^i - \Upsilon^i\left(k^i\right) - \tau^i b^i k^i - \tau_k^i k^i \\ &= \left[E^i\left(b^i\right) + Q^i\left(b^i, r\right) - 1 - \tau^i b^i - \tau_k^i\right] k^i - \Upsilon^i\left(k^i\right), \end{aligned}$$

where

$$\begin{aligned} E^i\left(b^i\right) &= \beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} \left(s v^i + t^i\left(b^i, s\right) - b^i\right) dF(s) \\ Q^i\left(b^i; r\right) &= \beta(r) \left(\int_{s^{i\star}(b^i)}^{\overline{s}} b^i dF(s) + \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} \left(\phi^i s v^i + t^i\left(b^i, s\right)\right) dF(s) \right). \end{aligned}$$

The solution to this problem yields a demand function for both credit and investment, given by $b^{i\star}\left(r, \tau_b^i, \tau_k^i\right)$ and $k^{i\star}\left(r, \tau_b^i, \tau_k^i\right)$. The solution for leverage is given byFOC's

$$\frac{dE^i\left(b^i\right)}{db^i} + \frac{\partial Q^i\left(b^i, r\right)}{\partial b^i} - \tau_b^i = 0,$$

where

$$\begin{aligned} \frac{dE^i\left(b^i\right)}{db^i} &= -\beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} \left(1 - \frac{\partial t^i}{\partial b}\left(b^i, s\right)\right) dF(s) \\ &= -\beta^i\left(1-\alpha_b\right)\left(1 - F\left(s^{i\star}\left(b^i\right)\right)\right) \end{aligned}$$

and

$$\frac{\partial Q^i\left(b^i,r\right)}{\partial b^i} = \beta\left(r\right)\left(\int_{s^\star(b)}^{\overline{s}} dF\left(s\right) + \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} \frac{\partial t^i}{\partial b}\left(b^i,s\right)dF\left(s\right) - \left(1-\phi\right)v^i s^{i\star}\left(b^i\right)f\left(s^{i\star}\left(b^i\right)\right)\right)$$
$$= \beta\left(r\right)\left(1-F\left(s^{i\star}\left(b^i\right)\right) + \alpha_b F\left(s^{i\star}\left(b^i\right)\right) - \left(1-\phi\right)v^i s^{i\star}\left(b^i\right)f\left(s^{i\star}\left(b^i\right)\right)\right).$$

The demand for capital is given by

$$k^{i\star}\left(r,\tau_b^i,\tau_k^i\right) = \frac{1}{a^i}\left[E^i\left(b^{i\star}\right) + Q^i\left(b^{i\star},r\right) - 1 - \tau_b^i b^{i\star} - \tau_k^i\right].$$

The indirect utility of these intermediaries is given by

$$V^i\left(r,\tau_b^i,\tau_k^i,T_0^i\right) = J^i\left(b^{i\star}\left(r,\tau_b^i,\tau_k^i\right),k^{i\star}\left(r,\tau_b^i,\tau_k^i\right);r,\tau_b^i,\tau_k^i\right) + T_0^i,$$

where

$$T_0^i\left(b^{i\star},k^{i\star}\right) = \tau_b^i b^{i\star}k^{i\star} - \tau_k^i k^{i\star}.$$

The expected bailout per unit of capital is given by

$$\mathcal{B}^i = \beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} t^i\left(b^i,s\right)dF\left(s\right) + \beta\left(r\right)\int_{\underline{s}}^{s^{i\star}\left(b^i\right)} t^i\left(b^i,s\right)dF\left(s\right),$$

and the total amount of bailout funds raised is

$$-\left(1+\kappa\right)\sum_i \mathcal{B}^i k^{i\star}.$$

Market clearing in the financing market yields the equilibrium interest rate $r^\star\left(\boldsymbol{\tau}_b^i,\boldsymbol{\tau}_k^i\right)$, which is defined as the solution to

$$\sum_i Q^i\left(b^{i\star}\left(r^\star,\tau_b^i,\tau_k^i\right);r^\star\right)k^{i\star}\left(r^\star,\tau_b^i,\tau_k^i\right) = Q_S\left(r^\star\right).$$

Following the same logic as in the proof of Proposition 1, it follows that

$$\frac{dV^i}{d\tau_b^j} = \frac{\partial J}{\partial r}\frac{dr}{d\tau_b^j} + \tau_b^i\frac{db^i}{d\tau_b^j}k^i + \tau_k^i\frac{dk^i}{d\tau_b^j}.$$

After assuming that i) k^i is predetermined and normalized to 1, and ii) $\alpha_s = \alpha_b = 0$, then $\mathcal{B}^i = \beta\left(r\right)\alpha_b b^i$ and we can express the distortion as

$$\delta_b^i = \left(1+\kappa\right)\frac{\partial \mathcal{B}^i}{\partial b^i}k^{i\star} = \left(1+\kappa\right)\beta\left(r\right)\alpha_b$$