Using Elasticities to Derive Optimal Bankruptcy Exemptions*

Eduardo Dávila†

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Abstract

This paper studies the optimal determination of bankruptcy exemptions for risk averse borrowers who use unsecured contracts but have the possibility of defaulting. In a large class of economies, knowledge of four variables is sufficient to determine whether a bankruptcy exemption level is optimal or should be increased or decreased. These variables are i) the composition of households' liabilities, ii) the sensitivity of the credit supply schedule to exemption changes, iii) the probability of filing for bankruptcy with non-exempt assets, and iv) the value given by households to a marginal dollar in different states, which can be mapped to changes in households' consumption. I recover empirical estimates of the sufficient statistics using U.S. data over the period 2008-2016 and find that increasing exemption levels improves overall welfare, although there is substantial variation in estimated welfare gains across U.S. states and income quintiles.

JEL numbers: D52, K35, D14

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†Yale University and NBER.

1 Introduction

The number of personal bankruptcy filings in the U.S. is on the order of 1 million per year, affecting roughly 1% of American households. A recurring policy question is whether bankruptcy procedures should be harsher or more lenient with bankrupt borrowers. The level of bankruptcy exemptions, which determines the amount of resources that a borrower is allowed to keep in bankruptcy, is a key determinant of the leniency of the personal bankruptcy system.[1] Although it is widely believed that some form of credit relief is desirable, existing research has not identified how to determine the optimal exemption.

The lack of a simple but general framework to assess the welfare impact of exemption changes likely explains why comparable countries and regions have substantially different exemption levels. The wide dispersion in exemption levels across U.S. states is an example of this phenomenon. For instance, as of 2016, the homestead exemption for a bankrupt household in Massachusetts is \$500,000, while the homestead exemption for an identical household in Alabama is \$30,000.

In this paper, I study the question of how to set bankruptcy exemptions optimally. Using a canonical equilibrium model of unsecured credit, I show that, in a large class of economies, a few observable or potentially recoverable variables are sufficient to determine whether a bankruptcy exemption level is optimal, or should be increased or decreased. I then measure such observables to provide empirical estimates of the marginal welfare gains or losses associated with changing exemption levels for households in different income quintiles across U.S. states. These empirical estimates suggest that increasing current exemption levels is associated with welfare gains of varying degrees across states and income quintiles.

I first illustrate the main results of the paper in a baseline two-period model in which identical risk averse households borrow from competitive lenders using a debt contract with an option to declare bankruptcy. In this environment, I show that the marginal welfare change induced by a change in the exemption level m – measured in date 0 dollars – corresponds to the following expression:

$$\frac{\frac{dW}{dm}}{u'(c_0)} = \frac{\partial q_0}{\partial m} b_1 + \pi_m \mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right]. \tag{1}$$

This characterization shows that an elementary tradeoff determines whether it is optimal to increase or decrease the exemption level. On the one hand, a one-dollar increase in the exemption level m changes the price of debt issued by households by $\frac{\partial q_0}{\partial m}$ (always negative), which, when multiplied by the amount of debt issued, b_1, exactly captures the marginal cost of a more lenient bankruptcy procedure. This formalizes the notion that high exemptions hurt intertemporal smoothing by restricting credit supply.

On the other hand, a one-dollar increase in the exemption level effectively increases the resources in the hands of bankrupt borrowers in those states in which they file for bankruptcy with non-exempt resources. Multiplying the probability of filing for bankruptcy with non-exempt resources, π_m, by the valuation of a marginal dollar over those states, $\mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right]$, exactly accounts for the marginal benefit of increasing exemptions. This formalizes the notion that high exemptions improve households' ability to insure. Importantly, all elements of Equation (1) have empirical counterparts, which opens the door to finding empirical estimates of marginal welfare changes.

[1] I describe the institutional environment surrounding consumer bankruptcy in the U.S. in the Appendix. Figure 5 documents the evolution of bankruptcy filings in recent years. The Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 is the last significant legislative change related to bankruptcy.

It may come as a surprise that the characterization of the marginal welfare change, $\frac{\frac{dW}{dm}}{u'(c_0)}$, does not explicitly incorporate households' behavioral responses regarding how much to borrow and when to default. This occurs because households borrow and default optimally. The same logic applies to other optimally chosen variables, including labor supply or education decisions. This is a central insight of this paper that has thus far been overlooked in the context of bankruptcy design.

Because all right hand side variables in Equation (1) are endogenous to the exemption level, I explore the mapping between primitives and welfare through the identified sufficient statistics. In particular, I numerically illustrate how higher levels of default deadweight losses make bankruptcy exemptions less attractive by increasing the marginal cost of increasing exemptions. Intuitively, high levels of deadweight losses make credit supply schedules more sensitive to changes in the exemption level, which makes high exemptions less desirable. I also illustrate how increasing households' income uncertainty makes bankruptcy exemptions more desirable by increasing the marginal benefit of increasing exemptions. Intuitively, higher uncertainty over future income realizations increases the probability of filing for bankruptcy, which increases the insurance value of higher exemption levels.[2]

Subsequently, I extend the characterization of the marginal welfare change to a dynamic model with general preferences, richer investment and borrowing opportunities, flexible income processes, and that features household heterogeneity. The insights from the baseline model extend to this more general environment after appropriately weighting a sequence of the same marginal cost and benefit terms identified in Equation (1). In particular, I show that the identified sufficient statistics accommodate rich specifications of households' income processes that may include permanent and transitory shocks, health or family shocks, and life cycle considerations, which previous literature has shown to be important determinants of households' behavior. I also show that the precise determination of households' bankruptcy decisions, often challenging to characterize, only affects welfare through the identified sufficient statistics. In the context of this general model, I discuss how to account for multiple relevant institutional features, including the possibility of filing for bankruptcy under Chapter 7 or 13, the role of informal bankruptcy, and the existence of filing fees, among others.

This paper builds on existing structural work, since normative statements require a well-defined microfounded model. However, the new approach developed in this paper is most powerful when used to conduct a direct measurement exercise. In Section 4, building on the general model, I combine data from multiple sources to construct credible empirical counterparts of the identified sufficient statistics for income quintiles across U.S. states in the period 2008-2016. In order to measure the marginal cost of increasing exemptions, I first estimate sensitivities of credit supply schedules to exemption levels by using variation in exemption levels over time for different states. I then combine the estimated sensitivities with measures of the level of unsecured personal debt and auto loans per income quintile. In order to measure the marginal benefit of increasing exemptions, I first estimate transition probabilities into bankruptcy (with non-exempt resources) for households in different quintiles. I then combine them with measures of households' relative valuations across states, calculated from consumption expenditure changes across states. Using a recursive approach, I combine the directly measured estimates of two-period flow gains/losses with estimates of the transition dynamics of households across different income quintiles to be able to compute marginal welfare gains in net present value terms that account for households' income dynamics.

[2]This result does not necessarily hold in a fully dynamic model, especially if the income uncertainty is transitory. This idea is emphasized in Livshits, MacGee and Tertilt (2007).

The baseline empirical estimates yield three main conclusions. First, increasing the level of exemptions is associated with welfare gains across all quintiles over all the states considered. This is due to the low estimated sensitivity of credit supply schedules to exemption changes, which makes marginal costs an order of magnitude smaller than benefits. Second, the marginal welfare gains are higher in southern states, which experience a large incidence of bankruptcies, feature a large share of bankrupt borrowers with non-exempt resources, and whose households' perceive transition towards low income states as costlier in terms of consumption. Third, the marginal welfare gains are higher in willingness-to-pay terms for households in middle to higher income quintiles. This occurs because these households value the insurance offered by exemptions in bad states more than households that do not have much to insure.

On average, households value a marginal increase in the exemption level of one-dollar at 0.48 basis points. This number should be interpreted as households' marginal willingness-to-pay for an exemption increase. That is, the net present value of the benefits associated with an increase in exemptions of \$50,000 is roughly \$240 for an average household, although there is substantial variation across states and income quintiles. In some states, this number can be close to \$2,000, while for others it is close to zero. Extrapolating to the U.S. as a whole, a general \$50,000 increase in exemptions is valued at \$30bn. I discuss in detail the sensitivity of the results to estimate choices and preference parameters.

Lastly, I show how to account for three additional channels that modify the characterization of marginal welfare changes. First, I consider households who make borrowing or default decisions under distorted beliefs or preferences. In this case, the welfare impact of an exemption change must account for the impact on households' borrowing or default decisions. Second, I show that, provided that lenders' profits are invariant to the exemption level, the sufficient statistics identified remain valid, regardless of whether lenders have market power. Otherwise, computations of welfare changes must account for the impact of exemption changes on lenders' profits. Third, I show that the general equilibrium impact of exemption changes on prices (pecuniary effects) and aggregate demand also have a first-order effect on welfare. Even though each extension introduces a new channel that needs to be accounted for when calculating marginal welfare changes, the main forces highlighted in this paper do not vanish and still provide a natural starting point to assess the desirability of varying exemptions.

In the Appendix, I describe examples of the general environment analyzed in the paper and study additional extensions, including long-term debt and pooling and exclusion of borrowers, among others.

Related Literature This paper contributes to several strands of the literature on bankruptcy. The theoretical results in this paper contribute to the literature on applications of general equilibrium with incomplete markets, which has studied the possibility of default in general environments. Zame (1993) and Dubey, Geanakoplos and Shubik (2005) are the first to formalize the core tradeoffs behind the design of bankruptcy systems.[3] They show that introducing a default option may be welfare improving in a model with incomplete markets since it creates new insurance opportunities by introducing contingencies into existing contracts. While these papers take exemptions/default penalties as predetermined, I focus on the optimal determination of exemptions. Interestingly, despite acknowledging in Section 7 of their paper that the optimal penalty associated with default is neither zero nor infinite, Dubey, Geanakoplos and Shubik (2005) do not pursue its optimal determination. It is well known that default is only beneficial when markets are initially incomplete. Allowing for default when agents can write fully state contingent

[3] A closely related literature studies the role of collateralized credit in general equilibrium environments. See, for instance, Geanakoplos (1997, 2003, 2010) and Fostel and Geanakoplos (2008, 2014).

contracts, as in Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000), or Chien and Lustig (2010), only restricts the contracting space, reducing welfare unequivocally.

Many recent contributions to the question of consumer bankruptcy have been quantitative. For instance, the papers by Chatterjee et al. (2007) and Livshits, MacGee and Tertilt (2007) – by now standard references – provide a careful quantitative structural analysis of unsecured credit and bankruptcy from a macroeconomic perspective. See Livshits (2015) for a recent survey. Because of their rich equilibrium features, these papers have evaluated numerically the welfare implications of alternative bankruptcy procedures. I take a complementary but different approach. As in this literature, I use a microfounded model to be able to make normative statements. However, I do not calibrate the primitives of a specific model. Instead, I seek to provide analytical insights and identify a small set of observables that must be measured to determine optimal exemptions within a large class of models.

Methodologically, this paper develops a sufficient statistic approach for the problem of bankruptcy and security design. It derives formulas for the welfare consequences of policies in terms of observables rather than primitives – see Chetty (2009) and Weyl (2015). An advantage of this approach is that it applies to a large class of models. Applications of this approach include Diamond (1998) and Saez (2001) – hence the title analogy – on income taxation, Chetty (2006) and Shimer and Werning (2007) on social and unemployment insurance, Arkolakis, Costinot and Rodríguez-Clare (2012) on the welfare gains from trade, Berger et al. (2017) on the impact of house prices changes on consumption, and Alvarez, Le Bihan and Lippi (2016) and Auclert (2016) on monetary policy. In the context of financial markets, most closely related is the work of Matvos (2013), who uses sufficient statistics to estimate the benefits of contractual completeness. See also Dávila and Goldstein (2018) on deposit insurance.

The empirical findings of this paper contribute to the growing empirical literature that studies the impact on bankruptcy with unsecured credit. Earlier work providing evidence on consumer bankruptcies includes Gropp, Scholz and White (1997), Sullivan, Warren and Westbrook (2001), and Fay, Hurst and White (2002). More recently, Mahoney (2015), Dobbie and Song (2015), Dobbie and Goldsmith-Pinkham (2014), Cerqueiro and Penas (2016), Albanesi and Nosal (2015), and Herkenhoff, Phillips and Cohen-Cole (2016) shed light on the impact of bankruptcy procedure on household decisions, including health insurance, consumption, labor supply, and borrowing choices. The interest rate regressions that recover the sensitivity of interest rates schedules to exemption levels are most closely related to the work of Severino and Brown (2017), who empirically identify the impact of exemption changes on the quantity of credit and interest rates. See White (2007, 2011) for recent surveys of this body of work.

Finally, since varying exemptions affects the ability to enforce contracts, this paper relates to the theoretical literature on optimal contract enforcement. Krasa and Villamil (2000) study optimal contracting in a costly state verification environment when enforcement is endogenous but costly. Koeppl (2007) studies an optimal dynamic risk sharing problem along the lines of Kehoe and Levine (1993), in which the degree of commitment to repaying loans is endogenously determined. Kovrijnykh (2013) studies the impact of partial commitment on allocations and welfare in a related environment. Koeppl, Monnet and Quintin (2014) jointly characterize the efficient choice of contract enforcement and the efficient allocation of capital in a production economy.

Outline Section 2 introduces the main results of the paper within the baseline model. Section 3 extends the main results to a general dynamic environment. Section 4 presents the empirical application, and Section 5 studies additional channels relevant to the design of optimal exemptions. Section 6 concludes.

All proofs and derivations are in the Appendix.

2 Baseline Environment

This section introduces the main results of the paper in the simplest possible model: a two-period environment in which risk averse households borrow from competitive lenders using a debt contract with an option to declare bankruptcy. To simplify the exposition, I proceed as if the model features an interior solution with positive borrowing and describe the pertinent regularity conditions in the Appendix.

2.1 Environment

There are two dates, denoted by $t = \{0, 1\}$. At date 1, there are a continuum of possible states $s \in [\underline{s}, \overline{s}]$ with cdf $F(s)$. There is a single consumption good (dollar) in this economy, which serves as numeraire. There is a unit measure of ex-ante identical households who borrow from a unit measure of lenders.[4]

Households Households, who are risk averse, have well-behaved preferences represented by the discounted sum of expected utilities of consumption, with a discount factor $\beta \in (0, 1)$. Formally, households' objective function corresponds to

$$u(c_0) + \beta \mathbb{E}[u(c_1(s))], \tag{2}$$

where $u(c_0)$ denotes the flow utility of date 0 consumption and $u(c_1(s))$ denotes the flow utility of date 1 consumption in state s.

At date 0, households decide how much to consume and how much to borrow. Households consume their initial endowment of n_0 dollars in addition to the amount they borrow from lenders using a defaultable non-contingent contract (a debt contract). Formally, households face the following date 0 budget constraint

$$c_0 = n_0 + Q_0(b_1, m), \tag{3}$$

where n_0 denotes households' initial endowment and $Q_0(b_1, m)$ denotes the total amount raised from lenders by a household who takes on debt with face value b_1 when the bankruptcy exemption level (described below) is m dollars. The schedule $Q_0(b_1, m)$, determined by lenders' as described below, can be written as the product of the face value of debt, b_1, and the price of debt, $q_0(b_1, m)$, as follows $Q_0(b_1, m) = q_0(b_1, m) b_1$. Note that the inverse of the price of debt, $\frac{1}{q_0(b_1,m)}$ represents an interest rate schedule. When choosing b_1, households understand that $Q_0(b_1, m)$ or, equivalently, $q_0(b_1, m)$, are functions of the promised repayment b_1.

At date 1, households receive a stochastic endowment $n(s) > 0$ that depends on the realization of the state s. After s is realized, households decide whether to satisfy their promised repayment b_1 or to default.[5] If households repay, they simply consume their endowment net of the repayment b_1. If households default, they keep their endowment up to the bankruptcy exemption of m dollars, while any

[4]Alternatively, the problem studied in this section can be interpreted as the problem of a single household who borrows from competitive lenders.

[5]The terms bankruptcy and default are synonyms in this paper. See Section 3 for an explicit distinction between formal bankruptcy and informal bankruptcy (default) and for how to interpret the model to account for Chapter 7 and Chapter 13 bankruptcies.

non-exempt resources, $\max\{n(s)-m,0\}$, are transferred to lenders. Formally, households' consumption after repayment ($\mathcal{N}$) and after default ($\mathcal{D}$), corresponds to

$$c_1^{\mathcal{N}}(s) = n(s) - b_1 \tag{4}$$

$$c_1^{\mathcal{D}}(s) = \min\{n(s), m\}. \tag{5}$$

Therefore, we can recursively express the problem faced by households at date 0 and their indirect utility for a given exemption m, $W(m)$, as follows

$$W(m) = \max_{b_1} u(c_0) + \beta \tilde{V}(b_1, m), \tag{6}$$
$$\text{s.t. Eq. (3)},$$

where $\tilde{V}(b_1, m) = \mathbb{E}[V(b_1, s, m)]$ and where $V(b_1, s, m)$ denotes the date 1 continuation value for a household with debt b_1 in state s for a given exemption m, defined as

$$V(b_1, s, m) = \max\left\{u\left(c_1^{\mathcal{D}}(s)\right), u\left(c_1^{\mathcal{N}}(s)\right)\right\}, \tag{7}$$
$$\text{s.t. Eq. (4) and (5)}.$$

Lenders The supply of credit is determined by the schedule $Q_0(b_1, m)$, which defines the total dollar amount that households receive from lenders at date 0 when they take a loan with face value b_1. Provided that $Q_0(b_1, m)$ is well behaved, the main results of the paper can be derived without further assumptions on lenders' behavior. However, to study a fully specified environment, I assume that lenders are risk neutral, perfectly competitive, and that they require a given rate of return $1+r^{\ell}$. Under these assumptions, $Q_0(b_1, m)$ corresponds to

$$Q_0(b_1, m) = \frac{\delta \int_{\mathcal{D}} \max\{n(s)-m, 0\} dF(s) + b_1 \int_{\mathcal{N}} dF(s)}{1+r^{\ell}}, \tag{8}$$

where $1-\delta$ denotes the deadweight loss associated with default and $\mathcal{D}$ and $\mathcal{N}$ respectively denote the default and no default regions, determined in equilibrium.

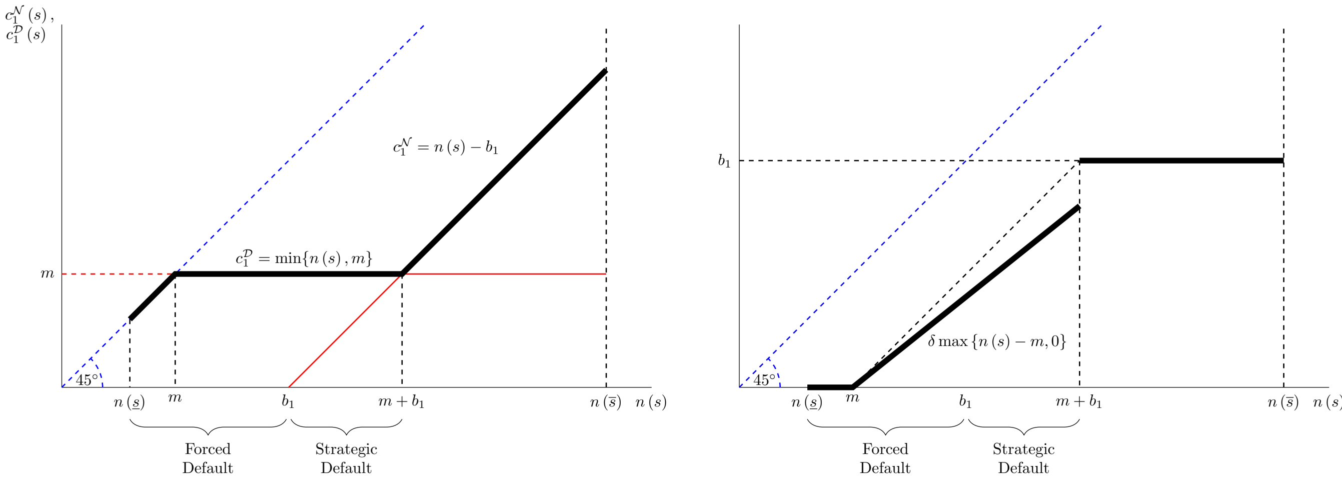
Equilibrium definition Given an exemption level m, an *equilibrium* is defined as a set of consumption and debt allocations c_0, $c_1(s)$, and b_1, default decisions, and a credit supply schedule $Q_0(b_1, m)$ such that i) households borrow and default optimally, internalizing the credit supply schedule, and ii) lenders' credit supply schedule satisfies a zero-profit condition.

2.2 Equilibrium Characterization

I sequentially characterize households' default decision, lenders' credit supply schedule, and households' borrowing choice. At date 1, households default whenever their consumption defaulting is higher than their consumption repaying. Formally, households default decision can be expressed as

$$\begin{aligned} &\text{if } n(s) - b_1 > m, && \text{Default} \\ &\text{if } n(s) - b_1 \le m, && \text{No Default.} \end{aligned} \tag{9}$$

As Figure 1 illustrates, households find optimal to default for low values of $n(s)$. This model incorporates both forced/involuntary default and strategic/voluntary default. Forced default happens



(a) Borrowers' date 1 consumption and bankruptcy decision

(b) Lenders' payment

Figure 1: Optimal default decision

Note: Figure 1a) and 1b) respectively illustrate the optimal default decision made by households and the repayment received by lenders for a given realization of households' date 1 endowment $n(s)$. The discontinuity in Figure 1b) is due to the presence of deadweight losses associated with default.

when a household does not have enough resources to fully pay back its debt, which occurs when $n(s) < b_1$. Strategic default happens when households have enough resources to repay their debt but decide not to do so, which occurs when $b_1 < n(s) < m + b_1$.

Given borrowers' default decision, lenders offer the following credit supply schedule in equilibrium

$$Q_0(b_1, m) = \frac{\delta \int_{\tilde{s}}^{\hat{s}} \max\{n(s) - m, 0\} \, dF(s) + b_1 \int_{\hat{s}}^{\overline{s}} dF(s)}{1 + r^{\ell}}, \tag{10}$$

where $\hat{s} = n^{-1}(m + b_1)$ denotes the threshold state that separates the default region from the no default region and $\tilde{s} = n^{-1}(m)$ denotes the threshold state that separates the partial recovery default region from the zero recovery default region. Figure 1 illustrates the repayment received by lenders. As shown in the Appendix, the credit supply schedule decreases with the exemption m, that is, $\frac{\partial Q_0(b_1,m)}{\partial m} = \frac{\partial q_0}{\partial m} b_1 \leq 0$, $\forall b_1$. This occurs for two reasons. First, an increase in m reduces lenders' recovery rates in default states. Second, an increase in m expands the default region, which increases deadweight losses associated with default. Both effects are passed to households through a lower $Q_0(b_1, m)$. I show that $Q_0(b_1, m)$ is increasing in b_1 when $b_1 = 0$, that is, $\lim_{b_1 \to 0} \frac{\partial Q_0(b_1,m)}{\partial b_1} > 0$, and single peaked, provided that the deadweight loss of default is non-zero.

At date 0, households optimally choose the equilibrium level of borrowing by solving

$$\max_{b_1} u(c_0) + \beta \tilde{V}(b_1, m),$$

where $c_0 = n_0 + Q_0(b_1, m)$ and

$$\tilde{V}(b_1, m) = \int_{\underline{s}}^{\hat{s}} u\left(c_1^{\mathcal{D}}(s)\right) dF(s) + \int_{\hat{s}}^{\overline{s}} u\left(c_1^{\mathcal{N}}(s)\right) dF(s),$$

where $\hat{s} = n^{-1}(m + b_1)$.[6] At the optimum, households' choose how much to borrow according to

$$u'(c_0) \frac{\partial Q_0}{\partial b_1} = \beta \int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s). \tag{11}$$

[6]To simplify the exposition, I proceed in this section as if $\underline{s} < \hat{s} < \overline{s}$. The Appendix includes a full characterization.

The left hand side of Equation (11) captures the marginal benefit of increasing the promised repayment b_1. It corresponds to the increase in resources available at date 0, $\frac{\partial Q_0}{\partial b_1} = q_0 + \frac{\partial q_0}{\partial b_1} b_1$, valued at households' date 0 marginal utility, $u'(c_0)$. The right hand side of Equation (11) captures the marginal cost of increasing the promised repayment. It corresponds to the discounted marginal utility of a dollar aggregated over the states in which households actually repay their debt.

Differentiating Equation (11) with respect to m determines how households' equilibrium borrowing reacts to changes in the exemption level. I show in the Appendix that the sign of $\frac{db_1}{dm}$, which is ambiguous, is determined by a combination of income, substitution, and direct effects. However, because households borrow optimally, the value of $\frac{db_1}{dm}$ is not necessary to compute the welfare impact of varying the exemption level, as shown next.

2.3 Main Results

Given that Equations (9), (10), and (11) fully characterize the equilibrium for a given exemption level, it is now possible to understand how changes in the exemption level m affect social welfare. Because lenders make zero profit in equilibrium, maximizing households' date 0 indirect utility $W(m)$, defined in Equation (6), maximizes social welfare in this economy. Proposition 1, which presents the central result of this paper, provides a test for whether to optimally increase or decrease the exemption level.

Proposition 1. (Directional test for a change in the exemption level m) *The normalized welfare change induced by a marginal change in the bankruptcy exemption m, expressed as a money-metric in date 0 dollars, is given by:*

$$\frac{\frac{dW}{dm}}{u'(c_0)} = \frac{\partial q_0}{\partial m} b_1 + \pi_m \mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right], \tag{12}$$

where $\pi_m = \int_{\tilde{s}}^{\hat{s}} dF(s)$ denotes the probability of filing for bankruptcy while claiming the full exemption, $\hat{s} = n^{-1}(m + b_1)$ denotes the threshold that separates the bankruptcy region from the no-bankruptcy region, $\tilde{s} = n^{-1}(m)$ denotes the threshold that separates the region in which households claim the full exemption from the region in which they do not, and $\mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right] = \beta \int_{\tilde{s}}^{\hat{s}} \frac{u'\left(c_1^{\mathcal{D}}(s)\right)}{u'(c_0)} dF(s) / \int_{\tilde{s}}^{\hat{s}} dF(s)$ denotes the conditional expected value as of date 0 of a claim that pays a dollar in states in which bankrupt households claim the full exemption.

Proposition 1 formalizes the tradeoff between intertemporal smoothing and insurance behind the optimal design of bankruptcy procedures. On the one hand, an increase in the exemption level m makes households' borrowing more expensive through a reduction in the price of the debt issued, $\frac{\partial q_0}{\partial m} < 0$ (equivalently, through an increase in the interest rate charged by lenders). This increase in borrowing costs captures the marginal cost of a more lenient bankruptcy procedure and formalizes the notion that high exemptions hurt intertemporal smoothing by restricting credit supply through the response of lenders.

On the other hand, an increase in the exemption level m increases the resources that bankrupt households are allowed to keep when claiming the full exemption, that is, in states in which they file for bankruptcy with non-exempt resources. From an ex-ante perspective, households value these additional resources by combining the probability of declaring bankruptcy with non-exempt resources, π_m, with the conditional expected value of a marginal dollar in those states, $\mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right]$, according to their own stochastic discount factor. The households' valuation of the additional dollar of resources kept in bankruptcy captures the marginal benefit of a more lenient bankruptcy procedure and formalizes the notion that high exemptions improve households' ability to insure.

Two properties of the environment considered here that are crucial to derive such simple characterization. The first property is that households borrow and default optimally. This fact allows us to set to zero the welfare impact of exemption changes on the total amount borrowed $\frac{db_1}{dm}$ and the probability of bankruptcy $\frac{d\pi^{\mathcal{D}}}{dm}$, since these have a second-order impact on households' welfare. The second property is that lenders' welfare is invariant to exemption changes. This fact eliminates the need to account for the direct effect of exemption changes on lenders' welfare. Therefore, the sensitivity of the credit supply to exemption changes is left to internalize the impact of changes in lenders' behavior on households' welfare. For instance, the value of deadweight losses associated with bankruptcy is a direct input into the determination of $\frac{\partial q_0}{\partial m}$, as well as any other factor that determines lenders' credit supply decision.

Several insights emerge from the characterization established in Proposition 1. The following corollary, which provides the foundation for the empirical application in Section 4, formulates a central insight of the paper.

Corollary 1. (Sufficient statistics) *Proposition 1 implies that measures of four observable or recoverable variables are sufficient to determine whether a bankruptcy exemption level is optimal or should be increased or decreased. These variables are i) households' debt position, b_1, ii) the sensitivity of the credit supply schedule to a change in the exemption level, $\frac{\partial q_0}{\partial m}$, iii) the probability of filing for bankruptcy while claiming the full exemption, π_m, and iv) the conditional expected value of a dollar, using the households' stochastic discount factor, in states in which bankrupt households claim the full exemption, $\mathbb{E}_m\left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)}\right]$.*

While the abstract tradeoff between intertemporal smoothing and insurance can be traced back to Zame (1993) and Dubey, Geanakoplos and Shubik (2005), Corollary 1 shows that it is sufficient to observe a small number of variables to exactly quantify that tradeoff. Equation (12) provides a simple test, expressed as a function of potentially observable variables, for whether it is optimal to increase or decrease the bankruptcy exemption, starting from a given level. Section 3 shows that a modified version of Equation (12) that accounts for dynamics and heterogeneity in a more general environment shares the same set of sufficient statistics.

It is worth highlighting that the only sufficient statistic that needs to be computed by running a regression is the sensitivity of the credit supply to exemption changes. The remaining variables can be directly measured in the data without any transformation. The key object of interest to compare the marginal benefit from increasing m, which accrues at date 1, to the date 0 cost, is the households' valuation of resources in bankruptcy states relative to borrowing states. Under CRRA preferences, measures of consumption in bankruptcy relative to measures of consumption when borrowing are sufficient to construct such valuation measures.

The value of the marginal welfare change $\frac{dW}{dm}$ is exact and that does not contain any approximation error. In order to assess the welfare impact of non-local changes, one can integrate over the relevant exemptions. Formally, an increase the exemption level from m_0 to m_1 is associated with a welfare change given by

$$W(m_1) - W(m_0) = \int_{m_0}^{m_1} \frac{dW}{dm}(\tilde{m})\, d\tilde{m},$$

where $\frac{dW}{dm}(\tilde{m})$ is given by

$$\frac{dW}{dm} = u'(c_0)\frac{\partial q_0}{\partial m}b_1 + \beta\int_{\tilde{s}}^{\hat{s}} u'\left(c_1^{\mathcal{D}}(s)\right)dF(s) \tag{13}$$

and defined for each exemption level $\tilde{m} \in [m_0, m_1]$. Similarly, we can characterize the fixed point that determines $m^\star$ by setting $\frac{dW}{dm} = 0$.[7] For instance, in the baseline model, the optimal exemption satisfies

$$m^\star = \frac{\frac{\Pi_m\left\{c_1^{\mathcal{D}}\right\}}{c_0}}{\Lambda\frac{\partial\log(1+r)}{\partial m}}, \tag{14}$$

where $\Lambda \equiv \frac{q_0 b_1}{n_0 + q_0 b_1}$ and $\frac{\Pi_m\left\{c_1^{\mathcal{D}}\right\}}{c_0} \equiv \int_{\tilde{s}}^{\hat{s}} \frac{c_1^{\mathcal{D}}}{c_0}\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)}dF(s)$. However, it is not possible to solve for the optimal exemption level $m^\star$ as a function of primitives. Instead, by repeatedly applying Equation (13), under appropriate regularity conditions, one would eventually find the optimal bankruptcy exemption level in a given economy. Although the equation characterizing $m^\star$ must hold at the optimum, it does not provide a characterization of $m^\star$ as a function of primitives, because all right hand side variables are endogenous to the exemption level. Alternatively, one could make direct assumptions on how the sufficient statistics vary with m and then solve for a fixed point. Consistently with the empirical implementation in Section 4, I focus on marginal welfare changes, which can be credibly measured.

Advantages of the approach Proposition 1 advances the understanding of how to set bankruptcy exemptions optimally in two ways. First, the sufficient statistics can be measured directly to construct empirical counterparts of the marginal welfare change $\frac{dW}{dm}$. This approach motivates the empirical application conducted in Section 4. Importantly, since the exact specification of many primitives becomes irrelevant provided that the sufficient statistics can be measured directly, finding credible estimates of these variables becomes the ideal approach to determine the optimal exemption level. My results highlight the importance of improving the measurement of the variables identified in Corollary 1. This is the form in which the results of the paper are more powerful, since they encompass a much larger class of models that could be characterized analytically or solved computationally.

Alternatively, the characterization of Proposition 1 can be used to discipline and improve the understanding of theoretical and structural modeling. This approach motivates the numerical simulation conducted in Section 2.4, which links primitives to the optimal exemption through the sufficient statistics identified in Corollary 1. In richer models, one could set the sufficient statistics as targets to discipline a model's calibration, which would guarantee that the model predictions for the welfare impact of exemption changes are locally exact.[8]

The main drawback of the approach developed in this paper when compared to solving a fully specified model is the inherently local nature of the measurement exercise. Although, as just described above, one could in principle construct measures of the relevant sufficient statistics for any value of the exemption level and integrate over them to find global results, this is challenging to do in practice. Consequently,

[7] The condition $\frac{dW}{dm} = 0$ is necessary but not sufficient to find the optimal exemption. As usual in problems with continuous distributions of shocks, the convexity of the planning problem is in general not guaranteed. I discuss sufficient conditions for convexity and show that the model is well-behaved numerically for standard primitives in the Appendix.

[8] Livshits (2015) describes existing normative structural work on bankruptcy by stating that: "*the welfare assessments of bankruptcy regimes and reforms have been quite wide-ranging*". There is scope to also use the sufficient statistics identified in this paper to distinguish the channels through which different quantitative papers have reached different conclusions.

extrapolating marginal welfare changes far away from observed exemption levels may require to calibrate a fully specified quantitative model.

2.4 Numerical Simulation: Behind the Sufficient Statistics

Because every element of Equation (12) is endogenous, it is not easy to establish a direct link between the primitives of the model and the desirability of increasing or decreasing the exemption level. Before generalizing the results and implementing them empirically, I numerically solve the baseline model for different parameter specifications. The goal of this exercise is to illustrate the link between different model primitives and sufficient statistics and how these influence the optimal exemption policy.[9] This exercise highlights how by providing an intermediate link between primitives and welfare assessments, the approach developed in this paper is helpful to improve our understanding of complex structural models.

Given the simplicity of the baseline model, it would be ill-advised to try to conduct a serious calibration. I present here instead a numerical illustration of the underlying relevant channels. A credible empirical application based on the sufficient statistics identified in the context of the general model is developed in Section 4.

To numerically solve the model, I assume that households have constant relative risk aversion utility (CRRA) preferences, that is, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with relative risk aversion coefficient $\gamma \equiv -c\frac{u''(c)}{u'(c)}$. I assume that the distribution $F(s)$ of date 1 endowments is a log-normal with parameters of the underlying normal denoted by μ and σ. Parameters values, chosen to best illustrate the qualitative features of the model, are included in the description of each figure.

Comparative Statics through Sufficient Statistics I include in Appendix G a graphical illustration of each of the relevant objects that determine the equilibrium, including lenders' credit supply and households' default and borrowing decisions, as well as social welfare. However, in the text, I focus on two illustrative comparative statistics exercises. First, I describe through the lens of Proposition 1 how changes in the level of deadweight losses $(1-\delta)$ affect the determination of the optimal exemption level. Second, I use the same approach to describe how changes in the volatility of households' date 1 endowments (σ) affect the determination of the optimal exemption level. It is possible to carry out similar comparative statics exercise for other parameters of the model.

The top plots in Figure 2 illustrate how the optimal exemption changes with the recovery rate δ (left plot) and the volatility of households' date 1 endowments σ (right plot).[10] Figure 2 shows that higher recovery rates δ (lower deadweight losses in bankruptcy) and more volatile endowments are associated with higher optimal exemptions. The bottom plots in Figure 2 show that households are worse off when deadweight losses are large (δ is low) and when volatility is high, for any value of exemptions. These plots show that there is greater scope for welfare improvements through exemption changes when deadweight losses are small and when volatility is large, a similar insight that can be drawn from the top plots in Figure 3. The bottom plots also illustrate the potential drawback of the local approach developed in this paper. For instance, when m is sufficiently low, the welfare function is close to flat, even though the exemption level is far from optimal.

[9] A reader mostly interested in the measurement of the sufficient statistics can jump to Section 4 at little cost.

[10] Given that the distribution of $n_1(s)$ is log-normal, varying σ also affects the mean of households' date 1 endowment, but this effect is quantitatively insignificant.

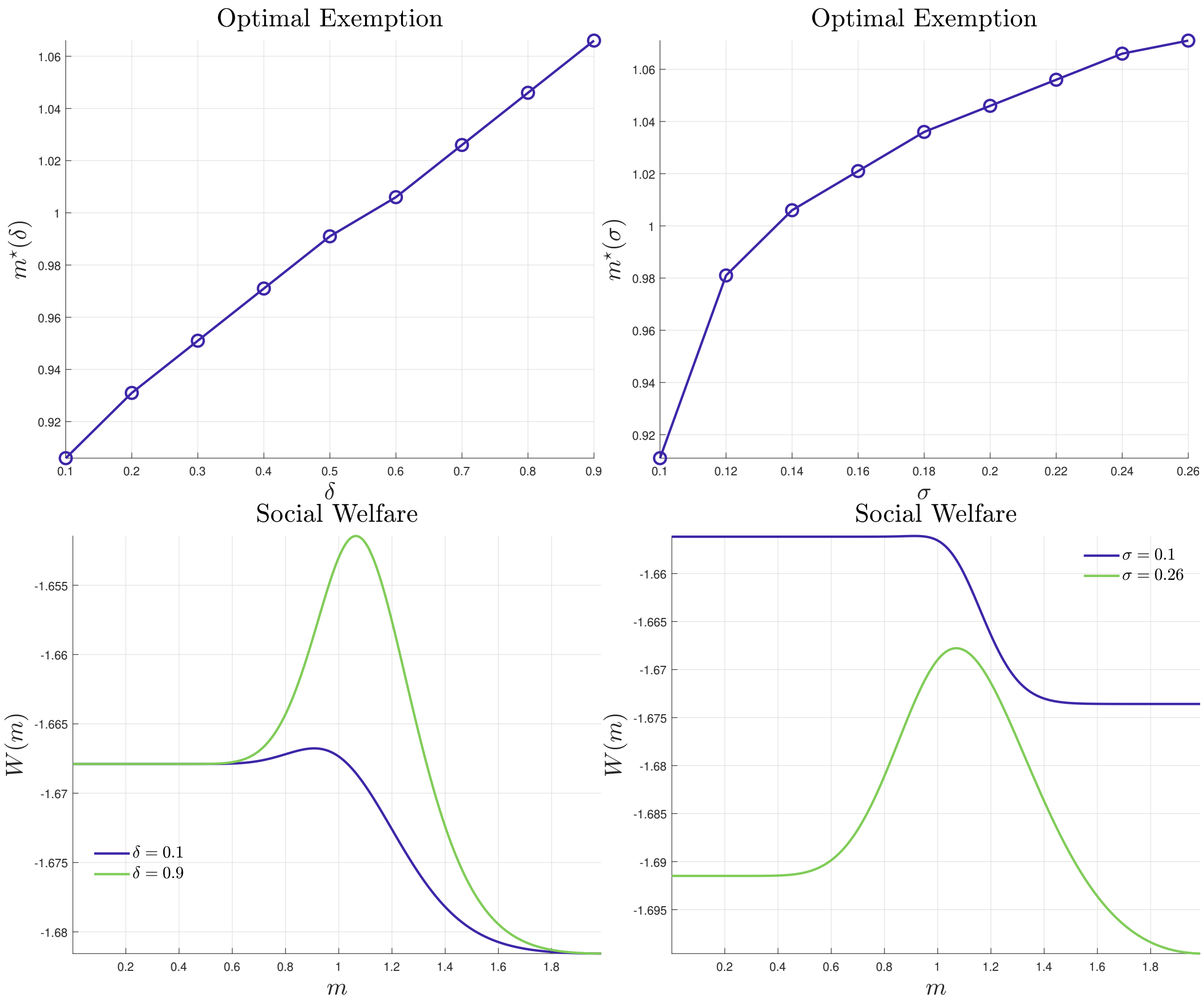


Figure 2: Comparative Statistics $m^\star(\delta)$ and $m^\star(\sigma)$

Note: The left panel of Figure 2 shows how the optimal exemptions $m^\star$, defined in Equation (14), varies as a function of the recovery rate by lenders in case of default δ (the complement of the dead-weight loss). The single period is meant to represent three years, with parameters $\beta = 0.92^3$, $r^\ell = 1.04^3 - 1$, $\gamma = 2$, $n_0 = 1$, $\mu = 0.15$, $\sigma = 0.18$, $\underline{s} = 0.2$, and $\overline{s} = 2$. I consider a range for the recovery rate δ given by $\delta \in [0.1, 0.9]$. The right panel of Figure 2 shows how the optimal exemptions $m^\star$, defined in Equation (14), varies as a function of the volatility of the distribution of date 1 endowments (σ). The single period is meant to represent three years, with parameters $\beta = 0.92^3$, $r^\ell = 1.04^3 - 1$, $\gamma = 2$, $\delta = 0.75$, $n_0 = 1$, $\mu = 0.2$, $\underline{s} = 0.2$, and $\overline{s} = 2$. I consider a range for the volatility given by $\sigma \in [0.1, 0.26]$.

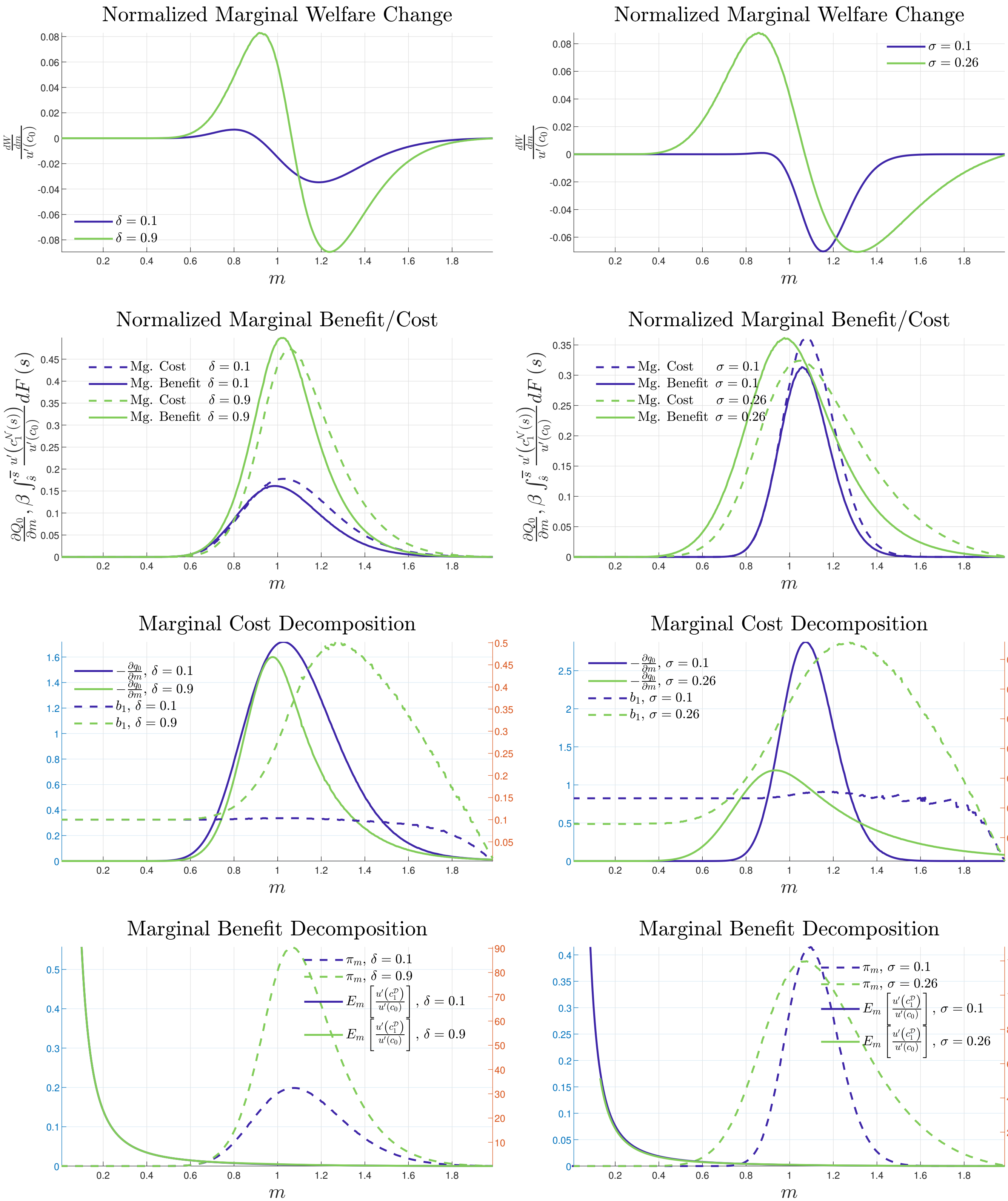


Figure 3: Marginal Welfare Change and Marginal Cost/Benefit Decomposition

Note: Each row of plots in Figure 3 respectively shows, from top to bottom: i) the normalized social welfare change as a function of m, defined in Equation (12), ii) the normalized marginal benefit, $\frac{\partial q_0}{\partial m} b_1$, and cost, $\beta \int_{\tilde{s}}^{\hat{s}} \frac{u'(c_1^{\mathcal{P}})}{u'(c_0)} dF(s)$, iii) the components of the normalized marginal benefit, $-\frac{\partial q_0}{\partial m}$ and b_1, and iv) the components of the normalized marginal cost, π_m and $\mathbb{E}_m\left[\frac{\beta u'(c_1^{\mathcal{P}})}{u'(c_0)}\right]$. All left figures have parameters $\beta = 0.92^3$, $r^{\ell} = 1.04^3 - 1$, $\gamma = 2$, $\delta = \{0.1, 0.9\}$, $n_0 = 1$, $\mu = 0.2$, $\sigma = 0.18$, $\underline{s} = 0.2$, and $\overline{s} = 2$. All right figures have parameters $\beta = 0.92^3$, $r^{\ell} = 1.04^3 - 1$, $\gamma = 2$, $\delta = 0.75$, $n_0 = 1$, $\mu = 0.2$, $\sigma = \{0.1, 0.26\}$, $\underline{s} = 0.2$, and $\overline{s} = 2$.

The next three rows of plots in Figure 3 allow us to decompose the marginal welfare change into marginal benefits and costs. Figure 3 shows that the credit supply is less sensitive to exemption changes everywhere when $\delta = 0.9$ relative to when $\delta = 0.1$. This should be intuitive, since when recovery rates are high, the total amount recovered by lenders after default varies less, reducing the sensitivity to exemption changes. This fact reduces the marginal cost of increasing the exemption level. Interestingly, since lenders pass the higher recovery rate to households through more attractive borrowing conditions, households borrow more in equilibrium (b_1 is higher when $\delta = 0.9$). On the one hand, the increase in borrowing increases the marginal cost of borrowing, but only substantially for high levels of m. On the other hand, the increase in borrowing, through the moral hazard effect, further increases the marginal benefit of borrowing, since households default more often. The marginal conditional expected valuation of a dollar, $\mathbb{E}_m\left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)}\right]$, is constant in both scenarios. Overall, the reduction in the magnitude of the credit supply sensitivity $\left|\frac{\partial q_0}{\partial m}\right|$ associated with high recovery rate δ dominates the other effects, pushing towards a higher optimal exemption level.

When considering variation in households' endowment volatility σ, the bottom plot shows that households default more often while claiming the full exemption when volatility is high. Defaulting more frequently increases the marginal benefit of increasing the exemption level for almost every value of m. On the other hand, when σ is high, households borrow less for low values of m, but at some point start to borrow more than when σ is low. It is also the case that the sensitivity of credit supply $\left|\frac{\partial q_0}{\partial m}\right|$ is higher and increasing in m for low-to-intermediate exemption levels, which increases the marginal cost of increasing exemptions. Overall, the increase in the marginal benefit of higher exemptions caused by the increase in bankruptcies dominates, making a higher exemption level optimal when households' endowment volatility is higher.

3 General Environment

This section shows that the insights derived from the baseline model described in Section 2 are robust and apply more generally. I now explicitly consider a dynamic environment in which a cross-section of households with time- and state-dependent utility make endogenous labor supply decisions and have access to a rich set of investment and borrowing opportunities. The environment considered here provides the foundation for the empirical application in Section 4.

3.1 Environment

Households There are $i = 1, \ldots, |I|$ different types of households in the economy. Every household lives for $T \leq \infty$ periods, where $t = 0, 1, \ldots, T$. Households can invest in a set J of assets, indexed by $j = 1, \ldots, |J|$. Households can also borrow using a set K of contracts, indexed by $k = 1, \ldots |K|$. Households, which maximize expected utility of consumption and leisure, have the option to declare bankruptcy, as described below.

It is convenient to formulate the households' problem recursively. I denote the set of endogenous state variables for a type i household by $\Psi_i = \{\boldsymbol{a}_i, \boldsymbol{b}_i, \boldsymbol{x}_i\}$, where $\boldsymbol{a}_i$ denotes a $|J| \times 1$ vector of assets (individual elements are denoted by a_i^j), $\boldsymbol{b}_i$ denotes a $|K| \times 1$ vector of liabilities (individual elements are denoted by b_i^k), and $\boldsymbol{x}_i$ denotes a vector of other endogenous non-financial state variables. The vector $\boldsymbol{x}_i$ may include households' human capital stock or a bankruptcy indicator, among other variables, and evolves according to a particular law of motion, which could be stochastic. I denote the set of exogenous state variables by $\boldsymbol{s}_i$, which follow a Markov process denoted by $F(\cdot|\Omega_i)$, where $\Omega_i = \{\Psi_i, \boldsymbol{s}_i\}$ denotes the set of endogenous and exogenous states. For simplicity, and consistently with the empirical application in Section 4, I exclusively consider one-period liabilities – see the Appendix for how to account for long-term debt. Each asset j has a payoff $z_j(\boldsymbol{s}_i)$, while every liability k is associated with an obligation $z_k(\boldsymbol{s}_i)$, which depends on the realization of the state $\boldsymbol{s}_i$. For instance, a debt contract can be modeled as a given liability with a constant repayment $z_k(\boldsymbol{s}_i) = \overline{z_k}$, $\forall \boldsymbol{s}_i$.

The flow utility of type i household in period t is given by $u_{i,t}(c_{i,t}, h_{i,t}; \Omega_i)$, where $c_{i,t}$ and $h_{i,t}$ respectively denote consumption and hours worked. The net income of a household of age t and type i is given by $y_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i) + w_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i) h_{i,t}$. The term $y_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i)$ corresponds to the net non-financial non-labor related income. If positive, it can correspond, for instance, to an inheritance, while, if negative, it can correspond to uninsured medical bills, legal bills, costs of divorce, or unplanned children expenses. The term $w_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i) h_{i,t}$ corresponds to households' labor income, where the wage $w_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i)$ can be a function of households' past choices through $\boldsymbol{x}_i$.

At the beginning of period t, each household decides whether to repay his liabilities or to default. We can express the value function of a type i household of age t at the stage in which the bankruptcy decision is made as follows

$$V_{i,t}(\Omega_i; m) = \max\left\{V_{i,t}^{\mathcal{D}}(\Omega_i; m), V_{i,t}^{\mathcal{N}}(\Omega_i; m)\right\}, \tag{15}$$

where the functions $V_{i,t}^{\mathcal{D}}(\cdot)$ and $V_{i,t}^{\mathcal{N}}(\cdot)$ respectively denote continuation values after declaring bankruptcy and after repaying outstanding liabilities. The value of repaying outstanding liabilities is given by

$$V_{i,t}^{\mathcal{N}}(\Omega_i; m) = \max u_{i,t}(c_{i,t}, h_{i,t}; \Omega_i) + \beta_i \mathbb{E}_t\left[V_{i,t+1}(\Omega_i'; m)\right], \tag{16}$$

subject to the following budget constraint

$$\sum_j a_{i,t+1}^j + c_{i,t} = y_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i) + w_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i) h_{i,t} + \sum_j z_j(\boldsymbol{s}_i) a_{i,t}^j - \sum_k z_k(\boldsymbol{s}_i) b_{i,t}^k + \sum_k q_{i,t}^k(\Psi_i', \Omega_i, m) b_{i,t+1}^k,$$

where $q_{i,t}(\Psi_i', \Omega_i, m)$, whose determination is described below, denotes the price of liability k. The value of declaring bankruptcy is given by

$$V_{i,t}^{\mathcal{D}}(\Omega_i; m) = \max u_{i,t}(c_{i,t}, h_{i,t}; \Omega_i) + \beta_i \mathbb{E}_t\left[V_{i,t+1}^{\mathcal{N}}(\Omega_i'; m)\right], \tag{17}$$

subject to the following budget constraint

$$\sum_j a_{i,t+1}^j + c_{i,t} = \mathcal{W}(m; \boldsymbol{a}_i, \boldsymbol{x}_i, \boldsymbol{s}_i) - f(\Omega_i), \tag{18}$$

where $\mathcal{W}(m; \cdot)$, which takes the form $\mathcal{W}(m; \cdot) = \min\{m, \circ\}$ with a second argument that is independent of m, denotes the net amount of wealth at the disposal of a household after bankruptcy, and $f(\Omega_i) \geq 0$ models a filing fee, meant to account for administrative and attorney fees. In principle Equation (18) allows bankrupt households to save or borrow after bankruptcy.

The dependence of $f(\Omega_i)$ on state variables captures that the complexity and cost of bankruptcy may vary with households' asset composition. The function $\mathcal{W}(m; \boldsymbol{a}_i, \boldsymbol{x}_i, \boldsymbol{s}_i)$ can take different forms depending on the type of exemptions considered. For instance, if m corresponds to a wildcard exemption that applies to a household's overall portfolio, $\mathcal{W}(\cdot)$ can be expressed as in Equation (19). Alternatively, if m corresponds to a homestead exemption that applies to a single asset (home equity, modeled here in net terms as asset $j=1$), $\mathcal{W}(\cdot)$ can be expressed as in Equation (20):

$$\mathcal{W}(m;\boldsymbol{a}_i,\boldsymbol{x}_i,\boldsymbol{s}_i)=\min\left\{m,\max\left\{y_{i,t}(\boldsymbol{x}_i,\boldsymbol{s}_i),0\right\}+w_{i,t}(\boldsymbol{x}_i,\boldsymbol{s}_i)\,h_{i,t}+\sum_{j=1}^{J}z_j(\boldsymbol{s}_i)\,a_i^j\right\}\qquad\text{(Wildcard)}\quad(19)$$

$$\mathcal{W}(m;\boldsymbol{a}_i,\boldsymbol{x}_i,\boldsymbol{s}_i)=\min\left\{m,z_1(\boldsymbol{s}_i)\,a_i^1\right\}.\qquad\text{(Homestead)}\quad(20)$$

I discuss below how this general environment captures many features considered to be relevant in the context of personal bankruptcy design.

Lenders The total amount of credit raised by a household i in period t is given by $\sum_k q_{i,t}^k(\Psi_i',\Omega_i,m)\,b_{i,t+1}^k$. As in the baseline model, as long as this object is well behaved, and lenders' profits are invariant to the exemption level, there is no need to make further assumptions on lenders' behavior. However, to study a fully specified environment, I assume that lenders are risk neutral, perfectly competitive, and that they require a given rate of return $1+r^\ell$. I also assume that there is a deadweight loss of $1-\delta$ per unit of resources transferred in bankruptcy. Under these assumptions, we can express the supply of credit available to household i as follows

$$\sum_k q_{i,t}^k(\Psi_i',\Omega_i,m)\,b_{i,t+1}^k=\frac{\delta\int_{\mathcal{D}_{t+1}}\mathcal{W}^{\mathcal{L}}(m;\boldsymbol{a}_i',\boldsymbol{x}_i',\boldsymbol{s}_i')\,dF(\boldsymbol{s}_i'|\Omega_i)+\int_{\mathcal{N}_{t+1}}\sum_k z_k(\boldsymbol{s}_i')\,b_{i,t}^k dF(\boldsymbol{s}_i'|\Omega_i)}{1+r^\ell},\qquad(21)$$

where $\mathcal{W}^{\mathcal{L}}(\cdot)$, which depends on the exact formulation of the bankruptcy procedure (e.g., homestead vs. wildcard, as described above), accounts for the repayment received by lenders in bankruptcy, and the sets $\mathcal{D}_{t+1}$ and $\mathcal{N}_{t+1}$ denote the default regions one-period forward, which are determined in equilibrium as a function of households choices Ω_i' and the realization of exogenous shocks. In order for Proposition 2 to be valid, it is important that lenders have the ability to condition their credit supply schedule on the relevant state variables Ψ_i' and that household i's credit supply schedules do not depend on the actions of other households, although credit supply schedules could depend on characteristics of the population of households.[11]

Equilibrium definition Given an exemption level m, an *equilibrium* is defined as a set of consumption, $c_{i,t}(\Omega_i;m)$, hours worked, $h_{i,t}(\Omega_i;m)$, assets, $\boldsymbol{a}_{i,t}(\Omega_i;m)$, liabilities, $\boldsymbol{b}_{i,t}(\Omega_i;m)$, other endogenous choices, $\boldsymbol{x}_{i,t}(\Omega_i;m)$, default decisions, and credit supply schedules $q_{i,t}^k(\Psi_i',\Omega_i,m)$, $\forall k$, for each household i such that i) households make optimal decisions, internalizing the credit supply schedules, and ii) credit supply schedules offered by lenders satisfy a zero-profit condition.

3.2 Main Results

The characterization of the equilibrium is similar to the baseline model. Given households' future default decisions, it is possible to determine credit supply schedules, which determine households' choices. Given the dynamic nature of the model, now households' bankruptcy and borrowing decisions have an important

[11]As shown in the Appendix, the set of sufficient statistics remains unchanged in an extension of the model that includes pooling of borrowers and exclusion. Intuitively, since pooled households only interact through the equilibrium credit supply schedule, the sensitivity of the credit supply schedule accounts for all the relevant information to assess the impact of exemption changes on households' welfare. See Section 5 for how to account for general equilibrium interactions.

forward-looking component. Instead of focusing on the positive predictions of the model, which does not have a tractable analytical representation and which would rely on numerical simulations, I directly introduce the counterpart of Proposition 1 in this more general environment.

Proposition 2. (Directional test for a change in the exemption level m: general model) *Let's denote by $W_{i,\tau} = V_{i,\tau}^{\mathcal{N}}(\Omega_i; m)$ the value function of a given household i that has not declared bankruptcy in period t. The welfare change induced by a marginal change in the bankruptcy exemption m for that household is given by*

$$\frac{dW_{i,\tau}}{dm} = \sum_{t=\tau}^{T} \beta_i^{t-\tau} \pi_{\tau,t}^{\mathcal{N}} \mathbb{E}_\tau^{\mathcal{N}} \left[\frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{\partial \sum_k q_{i,t}^k b_{i,t+1}^k}{\partial m} \right] + \sum_{t=\tau}^{T} \beta_i^{t-\tau+1} \pi_{\tau,t+1}^{\mathcal{D}} \mathbb{E}_\tau^{\mathcal{D}} \left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial \mathcal{W}_{t+1}}{\partial m} \right], \tag{22}$$

where $\pi_{\tau,t}^{\mathcal{N}}$ and $\pi_{\tau,t}^{\mathcal{D}}$ respectively denote the equilibrium repayment and default probabilities of the household in period t given period τ state variables, $\mathbb{E}_{\tau,t}^{\mathcal{N}}[\cdot]$ and $\mathbb{E}_{\tau,t}^{\mathcal{D}}[\cdot]$ denote the conditional expectations on repayment and default states of period t random variables given period τ state variables, and $\frac{\partial \mathcal{W}_{t+1}}{\partial m}$ denotes the partial derivative of $\mathcal{W}(m; \boldsymbol{a}_i, \boldsymbol{x}_i, \boldsymbol{s}_i)$ in period $t+1$.

Proposition 2 extends the results from the baseline model and shows that the nature of the sufficient statistics is the same in this more general environment. The marginal welfare change induced by a change in the exemption level is given by an appropriately weighted sum of the costs associated with the increase in rates over the set of states in which households are borrowers and the benefits in the form of higher resources available in the states in which households' file for bankruptcy with non-exempt resources. The derivation of Proposition 2 exploits the fact that $\frac{\partial \mathcal{W}}{\partial m}$ takes the value of one whenever bankrupt households claim the full exemption but is zero otherwise. This feature implies that households value a marginal increase in the exemption as a dollar only in those states in which they claim the full exemption, regardless of whether that dollar is consumed, saved, or invested.[12] Importantly, the nature of the sufficient statistics is the same as in the baseline model, although in a dynamic model it is necessary to also measure the likelihood of alternative future scenarios.

Several insights emerge by considering the more general case. First, the precise determination of the region(s) in which households decide to default does not change the optimal exemption formula, because households default optimally. In general, forward-looking households internalize the option value of waiting before defaulting, which crucially depends on whether shocks are temporary or permanent. These considerations, which prevent a simple characterization of households default decisions, do not modify the set of relevant sufficient statistics. Similarly, changes in hours worked and in other endogenous choice variables do not affect $\frac{dW_{i,t}}{dm}$. Second, all marginal benefits and costs associated with an exemption change are valued according to households' own stochastic discount factor. When households' flow utility is separable in consumption, measures of households' consumption along with preference parameters are sufficient to appropriately value costs and benefits over time and across states. Third, the response of credit supply schedules to exemption changes still contains the relevant information to assess the marginal cost of exemption changes. As I show in the Appendix, this is also the case in environments in which households may be pooled or excluded from borrowing markets. Finally, one could derive a formula for the optimal exemption level $m^\star$ that must be satisfied at the optimum, as in Equation (14). This expression

[12]This consideration can only arise in a dynamic model. It would be incorrect to value the marginal gain of increasing the exemption level by a dollar using the sensitivity of households' equilibrium consumption to exemption changes.

averages marginal benefits/losses across periods and states but, as in the baseline model, cannot be calculated using exclusively local information.

3.3 Additional Remarks

The environment studied in this section is rich enough to capture the most relevant institutional features for the determination of households' bankruptcy decisions, in particular, those often included in quantitative bankruptcy models.

General Preferences. Households' preferences and risk attitudes in the model are potentially time- and state-dependent. This generality accommodates multiple phenomena. First, households can experience non-pecuniary costs associated with bankruptcy, consistently with White (1998) and Fay, Hurst and White (2002). These authors find that the fraction of households that would benefit from bankruptcy, based on pecuniary considerations, far exceeds the fraction of households who actually file for bankruptcy. This discrepancy is often attributed to stigma, social pressure, or hounding by lenders. Second, households can derive non-pecuniary benefits by holding a given asset. This is particularly relevant in the context of housing, which is an asset with a non-negligible consumption dimension. Third, households preferences can depend on past variables, as in habit models. Overall, given the other sufficient statistics, preferences only affect the characterization of the marginal welfare change through their effect on how households value marginal gains and losses over time/across states. For instance, this model can account for a time-varying risk premium that weights marginal costs and benefits differently over the business cycle. To simplify the exposition, I have restricted my attention to expected utility preferences, although the Appendix includes an extension with Epstein-Zin preferences.

General Income Dynamics. The shocks that determine households' income processes can be mapped to health shocks, employment shocks, and family shocks (e.g., childbirths, spousal separations, etc.), which existing literature has found to be relevant drivers of borrowing and bankruptcy decisions. The model also allows for shocks to income to be permanent, transitory, or a mixture of both. This is important, since Livshits, MacGee and Tertilt (2007) show that the nature of income shocks — whether they are transitory or permanent — as well as life-cycle borrowing motives play a critical role on households decisions. Taxation and social insurance programs that affect borrowing and bankruptcy decisions can also be accommodated by reinterpreting $y_{i,t}(\boldsymbol{x}_i, \boldsymbol{s}_i)$. Finally, note that households in this model can invest in their own human capital, which can be mapped to an endogenous state variable in $\boldsymbol{x}_i$ that affects households' labor income.

Chapter 7 vs. Chapter 13. When a period in the model is sufficiently long, default in the model can be interpreted as a Chapter 7 or a Chapter 13 bankruptcy. As described in Appendix A, Chapter 7 bankrupt debtors give up all unsecured assets above predetermined exemption levels, in exchange for an immediate discharge of liabilities, while Chapter 13 debtors are able to keep their assets, in exchange for proposing a repayment plan. Importantly, the repayment plan must give creditors at least the same amount they would receive under Chapter 7, so the level of bankruptcy exemptions is a key determinant of Chapter 13 repayment plans. Therefore, if a period in the model is taken to be two years or longer, it is reasonable to abstract from modeling the exact details of monthly repayment plans, and simply interpret the function $\mathcal{W}(m; \boldsymbol{a}_i, \boldsymbol{x}_i, \boldsymbol{s}_i)$ as the amount of net wealth available to households after filing for bankruptcy and committing to their repayment plan. Not accounting for Chapter 13 bankruptcies would underestimate the marginal benefit of exemptions in the empirical application, since virtually all Chapter

13 bankruptcies are asset bankruptcies in which marginal changes to the exemption level should affect the marginal amount of wealth kept by households in bankruptcy.

Forced vs. Strategic Default. The distinction between forced default, in which households declare bankruptcy because they do not have enough resources to pay back, and strategic default, in which household declare bankruptcy despite having enough resources to pay back their debt, often plays a prominent role in informal discussions about bankruptcy. The results of this paper show that this distinction is not relevant for the ex-ante optimal determination of optimal exemptions. From a normative perspective, it is only relevant whether households default and whether they claim the full exemption, but not whether default is forced or strategic.

Informal Default/Renegotiation. Instead of formally filing for bankruptcy, debtors can cease payments without filing, entering into "informal bankruptcy" (Dawsey and Ausubel, 2004). If all parties anticipate and price the possibility of informal bankruptcy appropriately, and the law of motion of unpaid debts is independent of the exemption level, Proposition 2 would remain unchanged. Explicitly modeling a renegotiation process in which borrowers and lenders bargain over the terms of a new contract after default may require to account for the additional benefits of changing exemptions in those states, since households' outside option depends directly on the exemption level. Whether this is the case depends on the details of the bargaining protocol. If lenders have full bargaining power, one may conjecture that allowing for this form of bargaining is irrelevant for the results of the paper, but there is scope to study further how bargaining in informal bankruptcy affects the determination of optimal exemptions.

Filing Fees. Filing and legal fees can determine in practice whether a debtor files for bankruptcy and under which chapter. Albanesi and Nosal (2015) document that the increase in attorney costs caused BAPCPA reduced total filings and increased the share of Chapter 13 bankruptcies. The formulation of filing fees in Equation (18) does not explicitly model the possibility of failing to file for bankruptcy due to a lack of liquid assets. Explicitly modeling a law of motion for unpaid debts such that a household needs enough liquid assets to file for bankruptcy would not affect Proposition 2.

Contract Selection. This paper follows the tradition of general equilibrium with incomplete markets, imposing no restrictions on the shape of contracts used while taking the set of available contracts as a primitive. I extend the results to allow households to endogenize the shape of contracts in the Appendix. As long as households optimally choose the shape of the contracts traded or the choice of contracts is invariant to the exemption level (for instance, debt contracts are observed in high and low exemption economies), Proposition 2 remains valid.

Interpretation of Planning Problem. In practice, bankruptcy exemptions are set by jurisdictions, not by private parties. This may reflect the complexity of coordinating borrowers with multiple lenders, especially in the case of multiple contracts. The problem solved in this paper can be interpreted as that of a jurisdiction setting exemptions to maximize ex-ante social welfare. Given that lenders make zero profit, it can also be interpreted as the problem of borrowers who optimally choose exemption levels ex-ante under commitment.

4 Empirical Results

The approach developed in this paper to determine whether it is optimal to increase or decrease exemptions is most useful when combined with direct measures of the sufficient statistics identified. With that goal, I now merge multiple data sources to recover empirical estimates of the identified sufficient statistics. I proceed in three steps. First, I describe how to implement empirically the directional test of the general model derived in Section 3. Second, I describe the data sources used to measure each of the sufficient statistics. Finally, I proceed to measure and combine all sufficient statistics in order to report estimates of marginal welfare changes by state and income quintile.

The empirical estimates of the sufficient statistics identified in this paper using data between 2008 and 2016 imply that there are welfare gains from increasing exemption levels across all income groups in all considered U.S. states. This result is due to the fact that the estimated sensitivity of credit supply to exemption levels, which directly modulates the cost of increasing exemption levels, is an order of magnitude smaller than the insurance benefits in case of bankruptcy.

4.1 Empirical Implementation

The empirical implementation is designed to recover measures of marginal welfare changes induced by exemption changes in the cross-section of households for different states. To reach that objective, and disciplined by the available data, I now operationalize the theoretical results derived in Proposition 2 as follows.

First, I assume that households have identical time-separable CRRA utility of consumption with discount factor β and constant relative risk aversion coefficient γ, and that they face the same form of uncertainty. Second, I take $T \to \infty$, and assume that households' value function admits a recursive representation. Both assumptions, when combined with a discrete representation of households' state space, yield the following result.[13]

Proposition 3. (Empirical implementation) *Given households' discount factor β and risk aversion coefficient γ, it is possible to compute the $S \times 1$ vector $\frac{d\hat{W}}{dm} = \left(\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega^1)}{u'(c(\Omega^1))}, \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega^2)}{u'(c(\Omega^2))}, \ldots, \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega^S)}{u'(c(\Omega^S))} \right)'$ of normalized welfare changes for households that transition between S possible states as follows*

$$\frac{d\hat{W}}{dm} = A^{-1}F, \tag{23}$$

where F is a vector of dimension $S \times 1$ and A is a matrix of dimension $S \times S$, given by

$$\begin{aligned} &\textit{(Flow Vector)} \quad F = Q_m + \beta\, diag\left(\Pi_{\mathcal{N}\to m} \times G'_{\mathcal{N}\to m}\right) \\ &\textit{(Transition Matrix)} \quad A = \mathbb{I}_S - \beta\left(\Pi_{\mathcal{N}\to\mathcal{N}} \odot G_{\mathcal{N}\to\mathcal{N}}\right) - \beta^2\left(\Pi_{\mathcal{N}\to\mathcal{D}} \odot G_{\mathcal{N}\to\mathcal{D}}\right) \times \left(\Pi_{\mathcal{D}\to\mathcal{N}} \odot G_{\mathcal{D}\to\mathcal{N}}\right), \end{aligned}$$

where $Q_m = \left(\frac{\partial Q}{\partial m}(\Omega^1), \frac{\partial Q}{\partial m}(\Omega^2), \ldots, \frac{\partial Q}{\partial m}\left(\Omega^S\right)\right)'$ is a vector of dimension $S \times 1$ that measures marginal welfare costs, given by $\frac{\partial Q}{\partial m}(\Omega) = \sum_k \frac{\partial q^k}{\partial m}(\Omega)\, b^{k\prime}(\Omega)$, $\Pi = \{\Pi_{\mathcal{N}\to\mathcal{N}}, \Pi_{\mathcal{N}\to\mathcal{D}}, \Pi_{\mathcal{D}\to\mathcal{N}}, \Pi_{\mathcal{N}\to m}\}$, denotes a set matrices of dimension $S \times S$ that define transition probabilities between no default states ($\mathcal{N}$), default states ($\mathcal{D}$), and default states in which household claim the full exemption (m), while $G = \{G_{\mathcal{N}\to\mathcal{N}}, G_{\mathcal{N}\to\mathcal{D}}, G_{\mathcal{D}\to\mathcal{N}}, G_{\mathcal{N}\to m}\}$ denotes matrices of dimension $S \times S$ that define households' relative valuations between the same set of states.

[13] I denote standard matrix multiplication by $\times$, element-wise matrix multiplication (Hadamard product) by $\odot$, and matrix transposition by X', for a given matrix X. I denote an identity matrix of dimension S by $\mathbb{I}_S$.

Proposition 3 provides a recursive representation of normalized welfare changes for the general environment.[14] This recursive representation decomposes the welfare impact of changing exemptions into a flow vector F, which maps directly to the characterization of the baseline two-period model, and a transition matrix A, which accounts for dynamics and households' valuations across states.

Given Proposition 3, constructing empirical estimates of the normalized welfare changes boils down to finding empirical counterparts of the elements in A and F. The vector F exclusively depends on measures of marginal cost Q_m and on the benefits of declaring bankruptcy while claiming the full exemption one period forward, given by $\beta \text{diag}\left(\Pi_{\mathcal{N}\to m} \times G'_{\mathcal{N}\to m}\right)$, exactly as in the baseline model. The matrix A exclusively depends on valuation matrices G and transition probabilities Π, whose element-wise products can be interpreted as shadow Arrow-Debreu prices (risk-neutral probabilities) from the perspective of a given household.

The ideal dataset to implement Proposition 3 would contain detailed information regarding i) households' balance sheets, in particular different types of liabilities, ii) income and other relevant individual state variables, for instance, health, education status, or past default history, iii) expenditure measures, iv) whether a household files for bankruptcy with or without non-exempt assets, and v) interest rate schedules for all forms of credit given households' characteristics and how these depend on exemption levels. Only the sensitivity of credit supply schedules to exemptions requires information on lenders' behavior. The remaining sufficient statistics can be collected in longitudinal household surveys. Given the available data sources, I describe my approach to measurement next.

4.2 Measurement Approach and Data Description

Here I describe how to construct empirical counterparts of the sufficient statistics from existing data sources. I map a period in the model to a three-year period in the data, as in Livshits, MacGee and Tertilt (2007) and I restrict the analysis to the period 2008-2016. This choice is motivated by the availability of data, since the dataset with the universe of bankruptcy filings starts in 2008, and by the need to avoid the period surrounding the implementation of BAPCPA (2005).

I use households' income as the single state variable, classifying households by income quintiles in each state. While one could in principle use a multidimensional set of state variables that includes households' income, assets, liabilities, and other characteristics, this would reduce the number of observations per state, with the associated loss of statistical precision. Consequently, I estimate marginal cost and benefit measures by income quintile, as well as transition and valuation matrices between income quintiles. I define income quintiles by state using data from the 2013 five-year estimates of the American Community Survey (ACS).

As discussed above, specifying preference parameters is only relevant to discount over time and to account for different relative valuations across states. Only two parameters are required: households' discount factor β and households' risk aversion coefficient γ. Consistently with existing quantitative literature, I set $\beta = (0.94)^3$ and $\gamma = 2$ as baseline parameters, although I discuss the sensitivity of the results to both choices.

[14] I focus on recovering normalized welfare changes, $\frac{dV^{\mathcal{N}}}{dm}/u'(c)$. The purpose of normalizing $\frac{dV^{\mathcal{N}}}{dm}$ by households' marginal utility is to compute money-metric measures in dollars. That is, the vector $\frac{d\hat{W}}{dm}$ computes the net present values of marginal welfare benefits and costs in different states, according to households' own stochastic discount factor.

Table 1: Empirical Counterparts

Measured Variable	Data Source	Maps Into
Interest-Rate Sensitivity	RateWatch, Exemptions Dataset, Regional Controls (ACS/BLS/FHFA)	Q_m
Households' Liabilities	PSID	Q_m
Households' Expenditure	PSID	G
Income Transition Matrix	PSID	Π
Prob. of Bankruptcy	ID-FJC	$\Pi_{\mathcal{N}\to m}$, $\Pi_{\mathcal{N}\to\mathcal{D}}$
Prob. of No-Asset Bankruptcy	ID-FJC	$\Pi_{\mathcal{N}\to m}$

Note: Table 1 schematically describes the data sources used to calculate each sufficient statistic required to compute $\frac{d\hat{W}}{dm}$, as described in Proposition 3.

I briefly describe here the data used to construct measures of A and F, along with a few relevant sample restrictions. Appendix B contains a more detailed description of the sources of data, as well as summary statistics. I apply the same methodology to the 38 states whose homestead exemptions are not unlimited and whose state-level exemptions are not dominated by federal exemptions.

First, to compute default probabilities for households with different income levels and to calculate the fraction of no-asset bankruptcies state-by-state, I use data on the universe of bankruptcy filings provided by the U.S. Department of Justice, through the Integrated Database of the Federal Justice Center (ID-FJC). The ID-FJC contains information regarding households' income when filing for bankruptcy, which allows us to construct estimates of the probability of bankruptcy and no-asset bankruptcy by income quintiles for each state. This information serves as an input to calculate $\Pi_{\mathcal{N}\to m}$ and $\Pi_{\mathcal{N}\to\mathcal{D}}$.

Second, to estimate transition matrices across income quintiles at the state level, I rely on the Panel Study of Income Dynamics (PSID), a bi-annual longitudinal survey with information on expenditure, income, and wealth collected from a representative sample of households. I use the PSID waves between 2007 and 2015 to construct measures of liabilities (auto and personal loans) by income quintile for each state, which is a necessary input to compute Q_m. I also use the PSID to estimate by maximum likelihood a transition matrix between different income quintiles, which is a direct input into the calculation of all matrices in the set Π.[15] Finally, I use PSID's consumption expenditure data to construct the valuation matrices in the set G. The main limitation of the PSID is that it lacks information on whether households have filed for bankruptcy. For that reason, I estimate a single transition matrix between income quintiles, which I combine with the probability of bankruptcy and no-asset bankruptcy from the ID-FJC dataset to calculate transition matrices into bankruptcy with and without non-exempt assets, as described in the Appendix. Similarly, because I do not observe consumption measures conditional on bankruptcy, I estimate a single valuation matrix between income quintiles and states.

Third, to estimate the sensitivity of credit supply schedules to exemptions changes, I combine data from RateWatch, a data collection company that gathers quarterly interest rate data on different debt products from banks and credit unions at the branch level, with manually collected information on exemption levels over time. I also use data on income, unemployment, and house prices at the county level from the

[15] For simplicity, I estimate transition matrices only with observations of households who do not migrate between states in a given period. It is possible to extend the methodology to account for migration between states.

American Community Survey (ACS), the Bureau of Labors Statistics (BLS), and the Federal Housing Authority (FHFA) as regional controls. Given that the information on exemption levels is annual, I aggregate all variables to an annual frequency, so the units of observation are average yearly rates set by a given branch of a financial institution.

I proceed to recover measures of the sensitivity of credit supply schedules to exemption changes by running regressions of interest rates offered by banks to lenders on exemption levels and other controls.[16] The baseline empirical specification used to the recovered the desired sensitivity is

$$\log(1 + r_{it}) = \alpha_i + \alpha_t + \psi m_{st} + \theta X_{st} + \varepsilon_{ist}, \tag{R1}$$

where r_{it} denotes the net interest rate offered by banks to lenders in a given category, α_i denotes a branch fixed-effect, α_t denotes a time fixed-effect, m_{st} denotes the exemption level applicable in a given state in a given period, and X_{st} denotes regional controls. The use of branch-fixed effects for a fixed product category isolates changes in credit supply schedules, since this regression controls by the type and quantity of the loan and the time invariant lender characteristics, as emphasized below. The identification of the parameter of interest ψ is driven by changes in exemption levels over time. Severino and Brown (2017), who run similar regressions to study the impact of exemptions on total credit volume and interest rates over the period 1999-2005, provide empirical support to the hypothesis that exemption changes can be treated as plausibly exogenous.

Before describing the empirical estimates, it is important to discuss the conceptual link between the regression coefficient ψ and its model counterpart $\frac{\partial q^k}{\partial m}$ (equivalently $\frac{\partial \log\left(1+r^k\right)}{\partial m}$). First, the model implies that the relevant object of interest is the elasticity of credit supply, $\frac{\partial q^k}{\partial m}$, not the equilibrium response of the price of credit (interest rates), $\frac{dq^k}{dm} = \frac{\partial q^k}{\partial m} + \frac{\partial q^k}{\partial b^k}\frac{db^k}{dm}$, which would include the change in households' borrowing decisions, $\frac{db^k}{dm}$. Since Regression R1 is run for a given credit product, which fixes the amount of credit b^k offered to a given household with a given creditworthiness, the recovered estimate is exactly measuring a credit supply elasticity. Second, it is possible that a change in the exemption level changes the composition of borrowers – in terms of unobservables – that borrow using a specific product in a given branch. In other words, even though Regression R1 is run for a given credit product that conditions on borrowers' observed creditworthiness, the composition of the set of borrowers that use that product may still change, which would affect the rate offered by lenders. In the Appendix, I analyze an extension of the model with pooling and exclusion that shows that, in that case, the right credit supply elasticity to recover is the one that incorporates these selection effects. Intuitively, from the perspective of a given borrower, the ability to borrow is modulated by the change in interest rates, regardless of whether this change comes from pooling and selection on unobservables or not. Finally, note that the ideal empirical design to recover $\frac{\partial q^k}{\partial m}$ would directly observe the credit supply schedule offered by lenders to a given borrower, defined by a given set of state variables that includes individual characteristics and borrowing choices, and computes its slope in the direction of m.

Table 2 reports the outcomes of Regression R1 for three different dependent variables: interest rates on personal unsecured loans, interest rates on auto loans, and interest rates on mortgages. It shows that increases in homestead exemption levels are associated with increases in interest rates charged by

[16] These regressions describe for the first time the relation between interest rates and exemptions in the post-BAPCPA period. See Severino and Brown (2017) for a full empirical study of how total credit and interest rates vary with exemption changes before the implementation of BAPCPA, with an emphasis on justifying a causal interpretation of estimates.

Table 2: Credit Supply Sensitivity

	Dependent Variable: log(Gross Interest Rate)						
	Personal	Personal	Personal	Auto	Auto	Auto	Mortgage
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Exemption (homestead)	0.0071*		0.0044	0.0058**		0.0054	−0.0001
	(0.0041)		(0.0079)	(0.0025)		(0.0037)	(0.0009)
Exemption (personal)		0.0185			0.0240		
		(0.0538)			(0.0201)		
Unemployment Rate	0.0003	0.0003	0.0005	0.0003*	0.0003	0.0004*	0.00004
	(0.0004)	(0.0004)	(0.0004)	(0.0002)	(0.0002)	(0.0002)	(0.0001)
House Prices	−0.00002**	−0.00002**	−0.00002***	0.000005	0.000004	0.000002	0.000001
	(0.00001)	(0.00001)	(0.00001)	(0.000003)	(0.000003)	(0.000003)	(0.000001)
Income	0.0023	0.0024	0.000002	0.00004	0.0001	0.0008	−0.0007
	(0.0017)	(0.0017)	(0.0018)	(0.0009)	(0.0009)	(0.0011)	(0.0005)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Branch FE	Yes	Yes	No	Yes	Yes	No	Yes
County FE	No	No	Yes	No	No	Yes	No
Observations	16,255	16,255	16,255	18,349	18,349	18,349	7,732
Adjusted R^2	0.8348	0.8348	0.3614	0.8885	0.8884	0.4473	0.8452

Note: Table 2 reports the results of running Regression R1 using personal unsecured loans' interest rates, auto loans' interest rates, and mortgage rates (thirty-year fixed rate mortgage) as the dependent variable. Exemptions are measured in millions of dollars. Interest rates are yearly averages from RateWatch, county unemployment rate is from the BLS, county house prices are from the FHFA, and county median household income is from the ACS.
Standard errors, in parentheses, are clustered at the state level, which allows for serial correlation in the error term.
***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level.

lenders for personal unsecured loans and auto loans (columns 1 and 4), but not for mortgages (column 7). Columns 2 and 5 show that the effect on rates offered of changes in personal exemptions has the same sign and even higher point estimates as the effect of changes in homestead exemptions, but these are not statistically significant. Columns 3 and 6 show that including branch fixed effects instead of county fixed effects accounts for a substantial share of the total R^2. This finding implies that branch-specific characteristics explain a substantial part of the total variation in offered interest rates. Motivated by the finding that mortgage rates are not sensitive to exemption levels, I parsimoniously disregard mortgage liabilities when calculating marginal welfare changes. This is a natural finding, since mortgages are a form of secured credit whose pricing is less linked to the level of exemptions. This finding also motivates the use of one-period credit in most of the paper.

In terms of magnitudes, increasing the homestead bankruptcy exemption by \$100k is associated with an increase in the interest rate charged on unsecured credit of roughly 7 basis points, with a standard deviation of 4 basis points, as shown by the first column in Table 2. Increasing the personal exemption by \$100k is associated with an increase in rates on unsecured credit of roughly 19 basis points (second column), with a standard deviation of 54 basis points. In the context of auto loans, increasing the bankruptcy exemption by \$100k is associated with an increase in the interest rate charged of 6 basis points (fourth column), with a standard deviation of 2.5 basis points. The same increase for personal exemptions is associated with an increase in rates of 24 basis points, with a standard deviation of 20 basis points. It is natural to find a smaller quantitative effect in auto loans relative to unsecured credit, since these loans are partly collateralized. As I will show next, these estimates, which directly determine the marginal cost of changing exemption levels are quantitatively small relatively to estimate benefits of increasing exemptions. This would be the case even if we used estimates one or two standard deviations higher than the highest point estimates.[17]

Given the amount of data available, the main limitation of this approach to recover credit supply sensitivities is that it only yields a single set of credit supply sensitivities for all states and all income quintiles. Because, as I show next, the estimated sensitivities are an order of magnitude lower than needed to counterbalance estimated marginal benefits, the broader empirical conclusions of the paper are not affected by the lack of richer credit supply elasticity estimates.

It is worth making two final remarks on the empirical implementation. First, if households happened to trade assets that directly reveal their willingness to pay for a unit of consumption in bankrupt states, it would be possible to calculate valuation matrices without making any parametric assumption on households preferences. Second, having access to a longer time series would allow us to include richer life-cycle dynamics into the model – adding age as a state variable – as well as calculating more refined valuation matrices – for instance, including an aggregate state to account for time-varying risk premia.

4.3 Main Results: Estimates of Marginal Welfare Change

Following the procedure just described, I use Proposition 3 to combine estimates of flow vectors F and transition matrices A for each U.S. state. The baseline empirical findings are based on the point estimates for the credit supply sensitivities for personal unsecured credit and auto loans from Columns 1 and 4 of

[17] The fact that interest rates are not too sensitive to changes in the leniency of bankruptcy procedures is consistent with the aggregate time-series evidence generated by the introduction of BAPCPA, which does not show a sharp change in interest rates after its implementation.

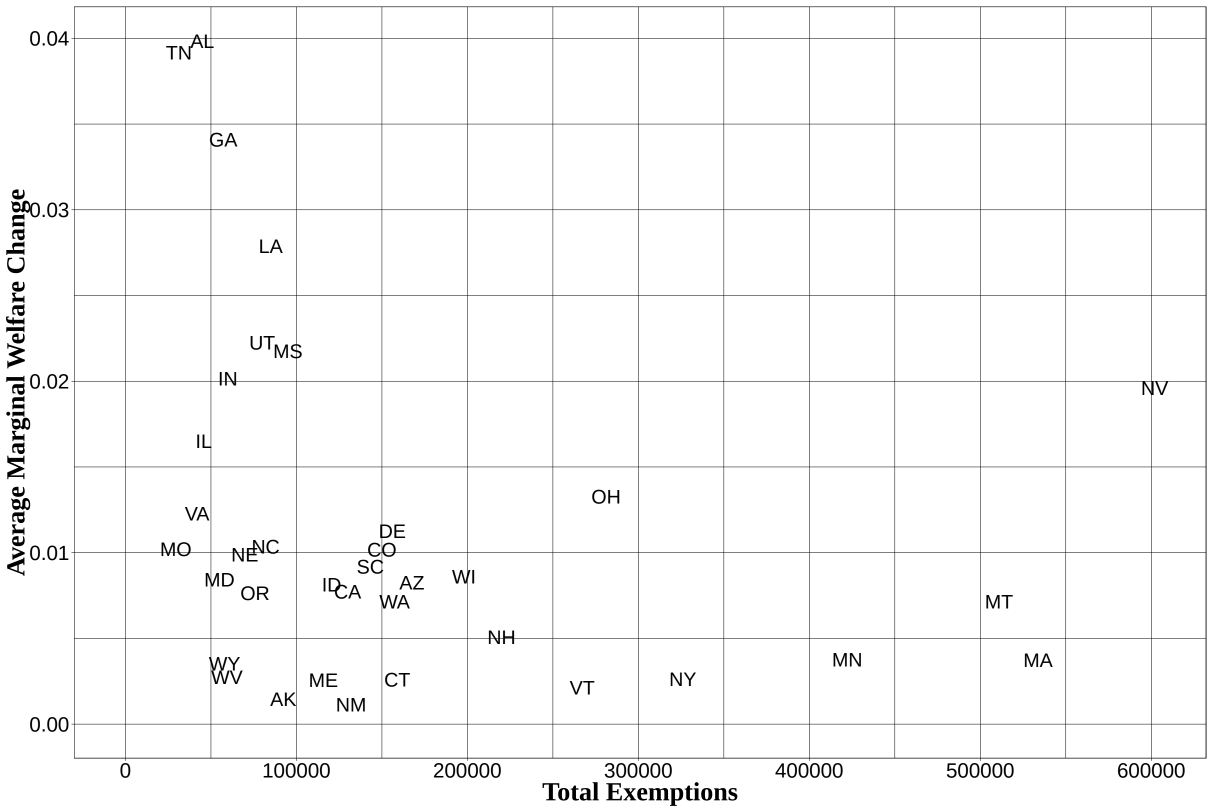


Figure 4: Marginal Welfare Changes and Exemption Levels

Note: Figure 4 shows average normalized marginal welfare changes associated with a one-dollar increase in the exemption level by state. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income quintiles, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$. Exemption levels are in dollars as of 2016.

Table 2. As described above, I use $\beta = (0.94)^3$ and $\gamma = 2$ as baseline preference parameters.

Figure 4 reports the average across income quintiles of the marginal welfare change associated with a one-dollar increase in the exemption level as a function of current exemption values state-by-state. Figure 4 implies that all states find it welfare improving on average to increase exemption levels, although there is substantial cross-sectional variation. The empirical relation between average exemption gains and current exemption levels turns out to be negative, with average welfare gains found to be larger in southern states (Tennessee, Alabama, Georgia, and Louisiana).

Figures 6 and 7 show that marginal welfare gains are indeed positive for all quintiles across all states.[18] In principle, we could have found negative marginal welfare changes for some quintiles or states. The explanation for the general desirability of increasing exemptions is that the estimates of credit supply elasticities seem to be an order of magnitude lower than the potential benefits associated with enjoying a marginal dollar when bankrupt. Two reasons explain why those southern states seem to benefit the

[18] Note that my conclusion is different from both Chatterjee et al. (2007) and Livshits, MacGee and Tertilt (2007), who find that less lenient bankruptcy procedures improve welfare. Overall, existing quantitative literature has found an ambiguous relation between leniency in bankruptcy and welfare. See Livshits (2015) for a recent survey that highlights how welfare assessments may be sensitive to model parametrizations.

Table 3: Marginal Welfare Changes

	$\frac{d\hat{W}}{dm}$ (Marginal Welfare Change)					F (Flow Change)				
Quintile	Mean	S.D.	Pctl(10)	Median	Pctl(90)	Mean	S.D.	Pctl(10)	Median	Pctl(90)
First	0.0038	0.0034	0.0009	0.0026	0.0079	0.0016	0.0015	0.0003	0.0011	0.0033
Second	0.0050	0.0046	0.0010	0.0038	0.0117	0.0023	0.0022	0.0004	0.0017	0.0055
Third	0.0044	0.0039	0.0013	0.0028	0.0102	0.0020	0.0018	0.0005	0.0014	0.0046
Fourth	0.0046	0.0038	0.0015	0.0034	0.0100	0.0019	0.0015	0.0006	0.0015	0.0043
Fifth	0.0063	0.0045	0.0021	0.0051	0.0129	0.0016	0.0011	0.0004	0.0013	0.0033
Overall	0.0048	0.0041	0.0012	0.0035	0.0112	0.0019	0.0017	0.0004	0.0013	0.0045

Note: Table 3 reports summary statistics of the distribution of normalized marginal welfare changes by state associated with a one-dollar increase in the exemption level. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income levels, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$.

most from increasing exemptions. First, the probabilities of filing for bankruptcy in those states are large. Second, the probabilities of filing an asset case (claiming the full exemption) in those states are substantially larger than in other states. Table 9 illustrates these differences comparing the states of Alabama and Massachusetts. It should not be surprising that exemption limits bind more often in states with low exemptions levels. A final mechanism that increases the estimated gains for those states is the larger than average disparity in consumption across income quintiles, which increases the value of insurance when including dynamic considerations.

Table 3 reports summary statistics of the distributions of marginal welfare changes quintile by quintile for flow welfare changes F and normalized welfare gains $\frac{d\hat{W}}{dm}$. As expected, normalized welfare gains are larger than flow measures, since they account for future gains. It is worth highlighting that the gains and losses vary across income quintiles. Interestingly, flow welfare gains tend to be higher for households in middle income quintiles, and marginal welfare gains are often higher for households in middle and high income quintiles. Two forces explain these results. First, households in lower quintiles are less likely to file for bankruptcy with non-exempt assets, so they do not benefit at the margin from having access to higher exemptions. This is also partly because they borrow less. Also, households in the higher quintiles unconditionally declare bankruptcy slightly less often. Second, middle and higher income households find large gains from insurance, given that they anticipate the possibility of moving towards a lower income quintile. On the contrary, households currently in the first quintile value less the insurance role of exemptions, since they do not have much to insure against: they are already in a bad scenario.

Quantitatively, the average estimate of normalized marginal welfare changes of 48 basis points implies that an increase in the exemption level of $\$50,000$ is associated with a welfare gain of roughly \$240 (roughly 0.4% of average household income in the U.S.). For households in the highest income quintile, this number can be as high as \$645 dollars in some states (90th percentile in the distribution of states), or as low as \$315 (10th percentile in the distribution of states). For households in the lowest income quintile, this number can be as high as \$395 (90th percentile) or as low as \$190 (10th percentile). In a few states, average normalized welfare gains can be substantial on average. In Tennessee and Alabama, a $\$50,000$ increase in exemptions is valued at roughly \$2000 (roughly 4% of average household income). These numbers should be interpreted as households' willingness to pay for a change in exemptions. Because households' incomes differ across quintiles, the relative amount of income that a household is willing to give up may still be higher for households in low income quintiles. In aggregate terms, a back-of-the-envelope calculation for the U.S. economy (with roughly 125 million households) implies that a $\$50,000$ increase in exemptions is valued at $\$240 \times 125m = \$30$bn. While the average estimates are non-negligible, these results suggest that some regions may greatly benefit from exemption changes.

Finally, I explore the sensitivity of the results to varying the inputs that go into calculating marginal welfare changes. First, I explore how the results vary when considering alternative values for credit supply elasticities. In Table 11 in the Appendix, I report summary statistics of the distribution of normalized marginal welfare changes when taking the highest point estimates for exemption sensitivities and augmenting them by one standard deviation. Even in that case, average marginal welfare changes remain positive, although they are smaller in magnitude. Second, I explore the role of risk aversion for the estimates. Values of γ lower than 2 yield similar estimates to the baseline parametrization. By assuming values of γ greater or equal than 2 – Table 13 sets $\gamma = 4$ – it is possible to find moderately larger measures of welfare gains associated with higher exemption levels. The impact of increasing risk aversion is stronger for households in high income quintiles, who have the possibility of experiencing large consumption drops. Finally, although I do not report specific results in the paper, varying the discount factor β within a reasonable range has barely any impact on the results.

5 Additional Channels

Three relevant channels not considered so far call for augmenting the characterization of marginal welfare changes. First, I consider the problem of unsophisticated households who make financial decisions under distorted beliefs. Second, I consider the possibility that lenders have market power. Finally, I show how to account for the general equilibrium impact of exemption changes.

Two main takeaways emerge from the study of these new channels. First, even though each extension calls for augmenting the expression to compute marginal welfare changes, the main forces highlighted in this paper do not vanish and still provide the starting point to study the optimality of changing exemptions. Second, in these environments, exemption changes must be considered along with other policies. In particular, policies that restrict financial decisions of unsophisticated households', that tax lenders' profits, or that force households to internalize pecuniary and aggregate demand externalities should be jointly implemented with exemptions changes. When those policies are in place, optimal exemptions can be set according to the marginal benefits and costs studied in this paper.

Belief Distortions Throughout the paper, both households and lenders hold correct beliefs about the uncertainty they face. While this is a natural starting point, households sometimes fail to make optimal financial decisions.[19] I now modify the baseline model to assume that households make borrowing choices

[19] See for instance Campbell (2016) on the importance of correcting for behavioral biases and Gennaioli and Shleifer (2018) for a compendium of theoretical and empirical work that highlights the importance of individual belief formation to understand individual and aggregate decisions. Dávila (2014) shows how to design corrective policies in the context of financial market trading when investors have distorted beliefs. Nakajima (2012, 2017) studies bankruptcy in models with

using a different set of beliefs from the planner who computes social welfare – similar arguments apply when households face preference distortions. Formally, households maximize

$$\max u\left(c_0\right)+\beta \tilde{\mathbb{E}}\left[u\left(c_1\left(s\right)\right)\right], \tag{24}$$

subject to date 0 and date 1 budget constraints, defined in Equations (3), (4), (5). The expectation $\tilde{\mathbb{E}}\left[\cdot\right]$ is taken over a perceived distribution of date 1 endowments $\tilde{F}\left(\cdot\right)$, with the same support as $F\left(\cdot\right)$, which denotes the distribution used by the planner to compute social welfare. For simplicity, I assume that lenders and the planner share the same set of beliefs and that there are no distortions at date 1. This guarantees that the planner and the households agree on when to default ex-post. Formally, social welfare from the planner's perspective corresponds to $W\left(m\right)$, defined as

$$W\left(m\right)=u\left(c_0\right)+\beta \mathbb{E}\left[u\left(c_1\left(s\right)\right)\right], \tag{25}$$

where consumption, borrowing, and default choices are selected by households. The only difference between Equations (24) and (25) is that households' and the planner assess differently the likelihood of date 1 states. Households optimally borrow according to $u'\left(c_0\right)\frac{\partial Q_0}{\partial b_1}=\beta\int_{\hat{s}}^{\overline{s}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)d\tilde{F}\left(s\right)$, while the marginal welfare change associated with varying exemptions from the planner's perspective is now given by

$$\begin{aligned}\frac{dW}{dm}&=u'\left(c_0\right)\frac{\partial q_0}{\partial m}b_1+\beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\left(s\right)\right)dF\left(s\right)\\&+\beta\underbrace{\left(\int_{\hat{s}}^{\overline{s}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)dF\left(s\right)-\int_{\hat{s}}^{\overline{s}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)dF^{i}\left(s\right)\right)}_{\text{Households' Belief Distortion}}\frac{db_1}{dm}\end{aligned} \tag{26}$$

Three conclusions can be drawn from Equation (26). First, the impact of an exemption change on borrowing decisions, $\frac{db_1}{dm}$, has now a first-order effect on welfare, since the planner perceives that households borrow too much or too little. Interestingly, only beliefs over states in which households repay determine the belief distortion.[20] Second, the planner would find optimal to correct households' borrowing choices, for instance, through a borrowing cap. In that case, Proposition 1 would remain valid, while the belief distortion in Equation (26) determines the Pigovian correction to households' borrowing. Finally, belief distortions that affect households' bankruptcy decisions or lenders' credit supply determination will have an independent impact on social welfare. These fascinating issues are outside the scope of this paper.

Lenders' Market Power Assuming perfect competition among lenders implies that there is no need to account for lenders' welfare separately. While perfect competition is a standard assumption in quantitative bankruptcy models – see, for instance, the arguments in Chatterjee et al. (2007) – there exists evidence consistent with the presence of market power in lending markets. See the classic contribution of Ausubel (1991) or more recent ones by Corbae and D'Erasmo (2019) and Scharfstein and Sunderam (2013).

It is possible to relax the assumption of perfect competition and to account explicitly for lenders' welfare. Formally, starting from the baseline model and preserving the risk neutrality assumption allows

temptation preferences.

[20] In the context of secured credit, Simsek (2013) and Bailey et al. (2018) have explored this non-trivial interaction between the shape of contracts, borrowers' beliefs, and borrowing theoretically and empirically.

us to map lenders' utility to the net present value of lenders' profits, which can be expressed as a function of households borrowing choices and the exemption level as follows

$$W^{\ell}=\Pi^{\ell}\left(b_1,m\right)=\frac{\delta\int_{\mathcal{D}}\max\left\{n\left(s\right)-m,0\right\}dF\left(s\right)+b_1\int_{\mathcal{N}}dF\left(s\right)}{1+r^{\ell}}-Q_0\left(b_1,m\right),$$

where $Q_0\left(b_1,m\right)$ denotes the total amount lent at date 0. Regardless of how $Q_0\left(b_1,m\right)$ is determined, social welfare, equal weighted in dollars, can now be calculated as

$$\frac{\frac{dW}{dm}}{u'\left(c_0\right)}+\frac{dW^{\ell}}{dm}=\frac{\partial q_0}{\partial m}b_1+\pi_m\mathbb{E}_m\left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'\left(c_0\right)}\right]+\underbrace{\frac{d\Pi^{\ell}}{dm}}_{\substack{\text{Change in}\\ \text{Lenders' Profit}}}. \tag{27}$$

where the new term simply accounts for how a change in exemptions affects the net present value of lenders' profits.

Two conclusions emerge when interpreting Equation (27). First, the crucial assumption made so far is that lenders profits are invariant to the level of m, not necessarily equal to zero.[21] For instance, fixed costs of entry or limit pricing strategies are consistent with lenders that make positive profits but whose profits are insensitive to the exemption level. Second, even when lenders make positive profits, the sensitivity of the credit supply schedule to exemptions remains as the relevant sufficient statistic. In practice, pass-through measures of how exemption changes translate into interest rates may be affected by the form of competition in the lending market, but the sensitivity of the credit supply schedule to exemptions still accounts for the impact of lenders' behavior on households' welfare.

General Equilibrium Externalities The paper has abstracted from the impact of exemption changes on aggregate variables, including asset prices, wages, and demand-determined output. In general, changing exemptions may affect the prices of goods and assets. These effects have first-order welfare consequences since, by construction, the economy features incomplete markets. After augmenting the baseline model to include an asset held by households which trades at a price p_0 and $p_1\left(s\right)$ at dates 0 and 1, it is possible to express the marginal welfare change associated with varying the exemption level as follows

$$\begin{aligned}\frac{dW}{dm}&=u'\left(c_0\right)\frac{\partial q_0}{\partial m}b_1+\beta\int_{\mathcal{D}_m}u'\left(c_1^{\mathcal{D}}\left(s\right)\right)dF\left(s\right)\\&\quad\underbrace{+u'\left(c_0\right)\frac{dp_0}{dm}\left(a_1-a_0\right)+\beta\int_{\mathcal{D}_y}u'\left(c_1^{\mathcal{D}}\left(s\right)\right)\frac{dp_1}{dm}a_1dF\left(s\right)+\beta\int_{\mathcal{N}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)\frac{dp_1}{dm}a_1dF\left(s\right)}_{\text{Pecuniary Effects}},\end{aligned} \tag{28}$$

where $\mathcal{D}_m$ denotes the set of realizations in which households default claiming the full exemption, D_y denotes the set of states in which default but do not claim the full exemption, and $\mathcal{N}$ denotes the no default states.

The pecuniary effects in Equation (28) are a manifestation of distributive pecuniary externalities, using the language of Dávila and Korinek (2018). Similar effects emerge under pecuniary externalities that operate through binding constraints, like collateral externalities. Likewise, the expression for the marginal welfare change would need to be augmented to account for aggregate demand externalities in environments with price rigidities and constrained monetary policy, as shown in the Appendix.

[21] One may conjecture that increasing exemptions improves households' outside option when bargaining with lenders. This may plausibly lower banks' profits and the social benefit from increasing exemptions. However, other mechanisms may be at play, so studying the response of lenders' profits to exemption changes becomes a relevant empirical question.

6 Conclusion

This paper develops a methodology, using measurable sufficient statistics, to assess the desirability of varying bankruptcy exemption levels in a wide variety of environments. Measures of households' debt, of credit supply elasticities to exemption changes, of the likelihood of declaring bankruptcy, and of the relative valuation of resources across states, which under standard preferences can be calculated using consumption changes, are sufficient to determine whether a bankruptcy exemption level is optimal for a given economy, or should be increased or decreased. Other features of the environment need not be specified once these variables are measured. Empirical estimates of the sufficient statistics identified in this paper using U.S. data between 2008 and 2016 imply that increasing existing exemption levels improves overall welfare, although there is substantial variation regarding the magnitude of potential gains across U.S. states and income quintiles. These estimates provide the first cross-sectional assessment of whether exemptions should be increased or decreased for different U.S. states.

The approach developed in this paper has the potential to spur further research on both quantitative structural modeling and microeconometric work on bankruptcy. Comparing the sufficient statistics identified in this paper among different quantitative models of bankruptcy can shed light on the channels through which different assumptions on primitives affect optimal exemptions in each model. Refining the measures of the sufficient statistics identified in this paper can improve the accuracy of welfare assessments. Looking forward, a better understanding of i) how belief distortions and other forms of household unsophistication affect financial decisions, ii) how exemption changes affect the competitive structure of the lending sector, and iii) how pecuniary and aggregate-demand externalities interact with exemption levels will allow us to better assess bankruptcy reforms.

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Appendix

A Institutional Background

Bankruptcy in the U.S. in modern times is shaped by the Bankruptcy Code, enacted by Congress in 1978 and amended multiple times since then. In the U.S., debtors can choose between two different personal bankruptcy procedures: Chapter 7, often called "liquidation" bankruptcy, and Chapter 13, often called "repayment" bankruptcy. Regardless of the chapter chosen, all collection efforts by creditors stop when an individual files for bankruptcy. See Skeel (2001) for a historical account of the evolution of the U.S. bankruptcy system and Claessens and Klapper (2005) for an overview of bankruptcy procedures around the world.

Under Chapter 7, bankrupt debtors give up all unsecured assets above predetermined exemption levels, in exchange for discharging most types of liabilities. There are certain types of liabilities that are rarely dischargeable in bankruptcy, notoriously student loans and tax obligations. After a Chapter 7 bankruptcy, debtors are not obliged to use future income to repay pre-bankruptcy debts, a feature often described as "fresh start". Although there are federal bankruptcy exemptions, states have adopted their own exemption levels, so bankruptcy exemptions vary widely across states as a result. In some states, debtors can decide whether to use federal or state exemption levels. The most significant exemptions are the wildcard, personal, and homestead exemptions.

Under Chapter 13, bankrupt debtors are able to keep their assets, but they must propose a repayment plan. Repayment plans typically involve using a proportion of the debtor's future income to repay outstanding debts monthly, usually over a three-to-five-year period. The law prescribes that the repayment plan must give creditors at least the same amount they would receive under Chapter 7, so the level of bankruptcy exemptions is very relevant for Chapter 13 bankruptcies. In practice, it is often the case that repayment plans involve monthly payments that exactly match the non-exempt amount.[22] Because of their complexity, attorney fees are higher in Chapter 13 cases relative to Chapter 7 cases. Porter et al. (2017) document that average attorney fees as of 2015 for Chapter 7 bankruptcies are roughly $\$1,200$, while Chapter 13 fees are on average higher than $\$3,000$, although Chapter 13 fees can be paid over time as part of the repayment plan.

The Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 is the last major change to the bankruptcy code. Most significantly, it introduces a "means test" that debtors must pass in order to file under Chapter 7. If a debtor's income is over a certain threshold, the debtor may not be eligible to file under Chapter 7, leaving only the possibility of filing under Chapter 13. This legislative change makes it harder for high-income debtors to take advantage of large exemption levels, forcing them

[22] A popular online resource explains how bankruptcy exemptions affect a Chapter 13 repayment plan as follows: *"First, it's important to understand that creditors must receive the same or more in a Chapter 13 case as they would in a Chapter 7 case. Therefore, if the Chapter 7 trustee would have been able to sell property for the benefit of creditors in a Chapter 7 case, then the creditors should at least get that same amount in a Chapter 13 case. So how do you figure out the amount creditors should receive? It's pretty easy actually. It's the value of the debtor's non-exempt property. A Chapter 13 debtor must pay to keep the property that would otherwise have been lost had the debtor filed for Chapter 7 bankruptcy."* Another popular source describes the role of exemptions under Chapter 13 as follows: *"Even though debtors do not actually turn over their property in a Chapter 13 case, exemptions are still just as important as they are in a Chapter 7 case in valuing and distributing assets to creditors to satisfy debt."*

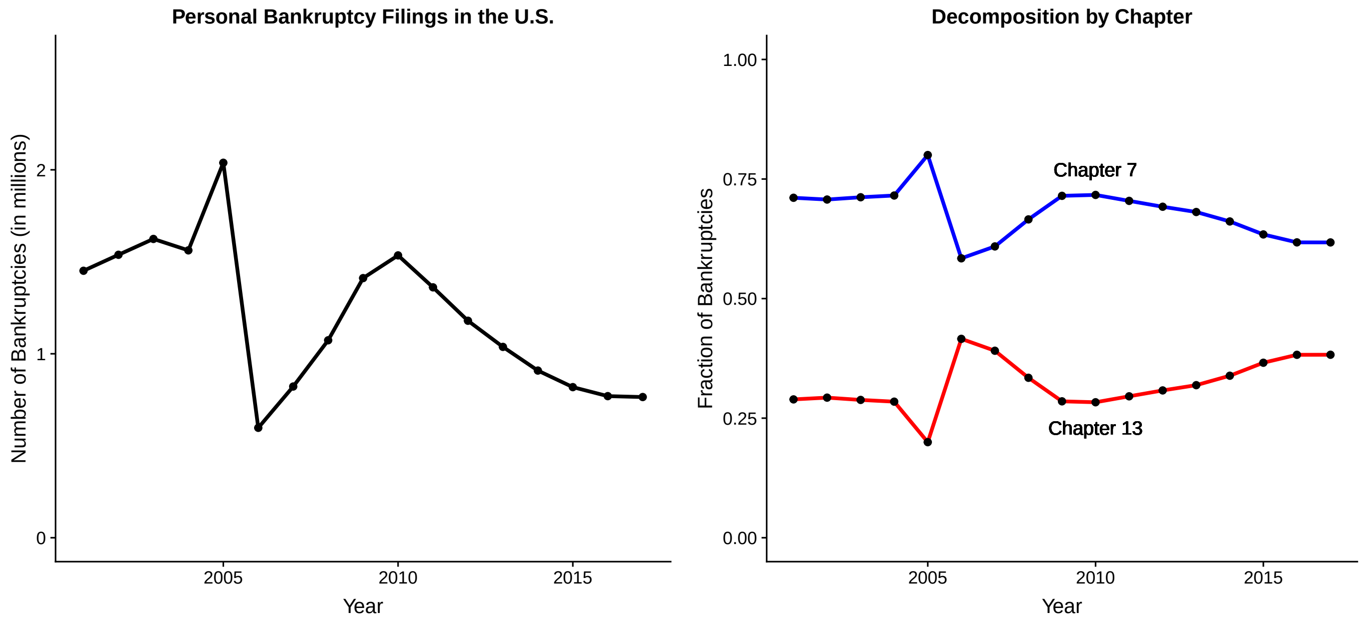

Figure 5: Evolution of Personal Bankruptcy Filings (2001-2017)

Note: The left panel of Figure 5 shows the yearly number of personal bankruptcy filings in the U.S. from 2001 until 2017, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center (ID-FJC). The right panel shows the fraction of Chapter 7 and Chapter 13 filings over the same period. See Livshits, MacGee and Tertilt (2010, 2016) for a systematic analysis of the long-run evolution of consumer bankruptcies, starting in 1970.

to file a repayment plan under Chapter 13. In addition to the means test, this reform also increases the filing fees and, through increased reporting requirements, the legal fees associated with filing for bankruptcy.

B Detailed Data Description

This Appendix provides additional discussion of the multiple sources of data used in the application in Section 4. Although the model should be interpreted in real terms, given the possibility of introducing biases by using inadequate price deflators and the minimal amount of inflation materialized over the period studied, I work with undeflated nominal variables. Similar results obtain when deflating all nominal variables with a consumer expenditure price index.

Bankruptcy Filings Data I use data from bankruptcy filings provided by the U.S. Department of Justice, through the Integrated Database of the Federal Justice Center (ID-FJC). This data and its associated documentation can be downloaded from https://www.fjc.gov/research/idb/. The ID-FJC database makes available *all* petitions filed under the Bankruptcy Code between October 2007 and September 2017. In addition to the date of filing, this dataset includes the filing chapter, the nature of the debt (consumer vs. business), the debtor's zip code, as well as measures of assets, liabilities, and monthly income, as reported by debtors when filing. Importantly for the approach developed in this paper, the dataset contains a variable (no-asset) that determines whether there will be funds available for distribution to creditors. Bankruptcy cases in which the value of non-exempt assets is less than the level of exemptions, so that there are no non-exempt resources left to pay unsecured creditors, are denominated "no-asset" bankruptcies.

Table 4: Bankruptcy Filings by Chapter and No-Asset Status

	Chapter 7	Chapter 13	Both Chapters
No-Asset Cases	67.55%	0.11%	67.66%
Asset Cases	3.12%	29.21%	32.33%
Both Cases	70.67%	29.32%	

Note: Table 4 reports the share of bankruptcies by chapter and by "no-asset" denomination in the U.S. from 2008 until 2016, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center. A bankruptcy filing is defined as an asset case when "there will be funds available for distribution to unsecured creditors, after any exempt property is excluded". As confirmed by personal communication with the ID-FJC database managers, it is likely that the negligible fraction of reported no-asset Chapter 13 bankruptcies are due to record-keeping inaccuracies, since Chapter 13 bankruptcies involve repayments to creditors.

Table 5: Summary Statistics of Bankruptcy Filing Information

	Mean	St. Dev.	Pctl(10)	Median	Pctl(90)	N
Net Worth	-73977	478464	-187425	-37537	12764	9383793
Assets	127948	287923	3720	65442	314232	9383793
Liabilities	201925	481458	24199	119837	441378	9383793
Income	41940	334532	4966	35578	79029	9383793

Note: Table 5 reports summary statistics on net worth, total assets, total liabilities, and yearly income of bankrupt filers in the U.S. from 2008 until 2016, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center. It provides information on the mean, median, standard deviation, as well as the 10th and 90th percentiles of each of the variables. All variables are measured in dollars and truncated to be smaller than $100 million.

I restrict the analysis to bankruptcies filed between January 2008 and December 2016 (nine calendar years). I restrict my attention to personal bankruptcies filed under Chapters 7 and 13, whose evolution is illustrated in Figure 5. Note that 96.5% of all bankruptcy filings in the U.S. are of personal/non-business nature. When using debtor's financial information, I disregard observations without zip code information, without no-asset information, and for which financial information (assets, liabilities, income) takes negative values. This process of data harmonization reduces the number of observations by roughly 9%, so the final dataset used to carry out the calculations in Section 4 and to construct Tables 4 and 5 contains 9.38 million bankruptcy filings.

Figure 5 shows the yearly number of total personal bankruptcies in the U.S., decomposed in terms of Chapter 7 and Chapter 13 bankruptcies. The increase in the number of bankruptcies between 2008 and 2010 is partly motivated by the experienced cyclical downturn and by the fact that there were an unusually large number of bankruptcies in 2005, right before the implementation of BAPCPA, which one can argue lowers the number of bankruptcies filed in 2008. After 2010, there is a downward trend in the number of personal bankruptcies in the United States. Most bankrupt borrowers file under Chapter 7, accounting for roughly 69% of filings over the period studied, as shown in Table 4. Figure 5 also shows that the share of Chapter 13 bankruptcies is steadily rising in the most recent years.

Table 5 shows summary statistics of assets, liabilities, and current income as reported by bankrupt debtors between 2008 and 2016. The measure of debtor's net worth is constructed as the difference between assets and liabilities. As expected, most bankrupt debtors have negative net worth.

Table 6: Summary Statistics of Bankruptcy Exemption Levels

	Mean	St. Dev.	Pctl(10)	Median	Pctl(90)	N
Homestead (2008)	113493	139083	10000	63750	300000	38
Personal (2008)	17641	10650	6280	15150	28945	38
Total (2008)	131134	141787	29290	88150	324080	38
Homestead (2016)	147499	151811	27893	97375	423000	38
Personal (2016)	24109	13292	11700	20300	35960	38
Total (2016)	171608	156038	45560	122250	448840	38

Note: Table 6 reports summary statistics for the level of personal, homestead and total state level exemptions (defined as the sum of personal and homestead exemptions) for the years 2008 and 2016. It provides information on the mean, median, standard deviation, as well as the 10th and 90th percentiles of the distribution of exemptions across states. Exemptions are expressed in dollars as of the relevant year, 2008 or 2016. The summary statistics include 38 states, but exclude the seven states (in addition to DC) for which homestead exemptions are unlimited and the five states in which households can opt for federal exemptions that are uniformly higher than state exemption levels.

Bankruptcy Exemptions Data I manually gathered and compiled a dataset with exemption levels by state over time. To do so, I relied on multiple editions of a popular bankruptcy filing book, Elias, O'Neill et al. (2015), as well as on state bankruptcy codes and press articles. This dataset contains homestead exemption levels and personal exemptions, the latter calculated by aggregating motor vehicle, wildcard, tools of trade, and cash or bank deposit exemptions. The dataset also includes whether bankrupt debtors in a given state have the option to use the federal limits, whether they can double their homestead or personal exemptions if married, and whether the applicable homestead exemption is unlimited. Since the majority of bankruptcies are filed jointly (roughly 70%, according to the ID-FJC), I use exemption levels for married couples in the empirical implementation in Section 4 and to construct Table 6.

Table 6 shows summary statistics of homestead, personal, and total bankruptcy exemptions across states in 2008 and in 2016. I exclude from the sample states with unlimited exemptions. I also exclude the five states (Hawaii, Kentucky, Michigan, New Jersey, and Pennsylvania) that allow debtors to choose between state and federal exemption levels and for whose state exemption levels are lower than federal exemption levels. Similar results obtain when one attributes the federal exemption level to these states. Figure 8 in the Online Appendix graphically illustrates the evolution of exemption levels over the period studied. The average level of exemptions moves from roughly \$113k to \$171k over the period, with homestead exemptions accounting for the major part of total exemptions.

Interest Rate Data The interest rate data necessary to find the sensitivity of credit supply schedules to exemption changes is gathered by RateWatch, a data collection company that tracks interest rate quotes at the branch level for a large number of financial institutions in the United States.[23] Both regulators (FDIC and Federal Reserve) and financial institutions regularly use this data to monitor the behavior of regulated entities or competitors. This is the only proprietary dataset used in the paper, and can be purchased through http://www.rate-watch.com/. The unit of observation are branch-level rates quoted for a specific loan product. I focus on interest rates quoted between 2008 and 2016 on unsecured personal loans, auto loans, and fixed-rate mortgages, since these loan classes map directly to the variables on debt

[23] This dataset also includes information on deposit rates, used for instance in Egan, Hortaçsu and Matvos (2017) and Drechsler, Savov and Schnabl (2017).

Table 7: Summary Statistics of Interest Rates and Regional Controls

	Mean	St. Dev.	Pctl(10)	Median	Pctl(90)	N
Personal Unsecured Rate	0.11	0.03	0.07	0.10	0.16	313799
Auto Loan Rate	0.04	0.02	0.02	0.04	0.06	207982
Mortgage Fixed Rate	0.04	0.01	0.04	0.04	0.05	26236
Unemployment Rate	6.85	2.53	4.20	6.50	10.00	32914
Median Income	51348.86	13064.15	39086.50	48927.00	66667.00	32914
House Price Index	387.86	213.78	182.62	352.81	645.85	32914

Note: Table 7 reports summary statistics (mean, median, standard deviation, as well as the 10th and 90th percentiles of each of the variables) for the levels of net interest rates offered across branches in the U.S. between 2008 and 2016 on personal unsecured loans, auto loans on two-year old cars, and thirty-year fixed-rate mortgages. It also presents summary statistics for the unemployment rate (BLS), median income (ACS), and the FHFA county house price index.

positions from the PSID. Table 7 shows summary statistics of quoted rates on unsecured personal loans, auto loans, and mortgages. As expected, average rates on personal unsecured credit rates are higher (11%) than average rates on auto and mortgage loans (4%).

Debt, Income, and Expenditure Data The empirical implementation of the results of this paper requires the knowledge of liabilities (personal debt, mortgage debt, auto debt) and expenditure information. The Panel Study of Income Dynamics is the one longitudinal study that contains detailed information on the required variables. I download information for the waves 2007-2015 and use households as the relevant unit of observation, automatically merging the different waves through the PSID Data Center data extraction procedure. I define total expenditure (consumption) as the sum of total food expenditure, utilities, telephone, and internet expenditure, transportation expenditure, education and childcare expenditure, and clothing, trips, and other recreation expenditure. I construct a measure of auto loans by combining loans on first and second vehicles. I construct a measure of personal loans by adding up the value of credit card, medical, legal, and other debt. I use total family income as the relevant income measure.

Table 8 shows summary statistics of the PSID information on households during the waves 2007, 2009, 2011, 2013, and 2015. Median yearly income is $48,000, with median expenditure given by roughly $22,000. I weight observations using the PSID core/immigrant family longitudinal weights. There is substantial variation in the number of available observations per state, with a median of 660 observations per state.

Regional Data The county-level controls used in Regression R1 come from different sources. Unemployment rates at the county level correspond to annual averages from the Local Area Unemployment Statistics released by the Bureau of Labor Statistics (BLS), which can be downloaded from https://www.bls.gov/lau/. House prices at the county level correspond to the House Price Index released by the Federal Housing Finance Agency, which relies on sales or refinancings of single-family homes whose mortgages have been purchased or securitized by Fannie Mae or Freddie Mac. This series can be downloaded from https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index.aspx. Income and population data at the county and state levels come from five-year estimates of the American Community Survey (ACS), which can be downloaded from https://factfinder.census.gov. While the other

Table 8: Summary Statistics of Households' Personal Information

	Mean	St. Dev.	Pctl(10)	Median	Pctl(90)	N
Income	67512	98266	10957	48050	136500	43673
Expenditure	26236	19776	7500	22360	47993	43673
Personal Debt	6961	41226	0	0	17000	43673
Auto Loans	4522	10087	0	0	18700	43673
Mortgage Principal	50272	98429	0	0	175000	43673

Note: Table 8 presents summary statistics for the sample of 43673 households obtained from the PSID over the period 2007-2015. All variables are measured in dollars for the current year. I restrict the sample to observations with annual expenditure greater than \$100 and negative income values smaller in magnitude than \$100,000. I weight observations using the PSID core/immigrant family longitudinal weights.

series are available from 2008 until 2016, the series for income at the county level is only available starting in 2010, which reduces the number of available observations for the regressions when income measures are added as a control. Table 7 shows summary statistics of all the variables.

C Empirical Results

This section contains two sets of results. First, it includes measures of bankruptcy probabilities by income level for two representative states: Alabama and Massachusetts. It also includes an average valuation matrix, to give a sense of the magnitudes involved. Second, it includes two figures representing marginal normalized welfare changes, $\frac{d\hat{W}}{dm}$, in Figure 6, and normalized flow welfare changes, F, in Figure 7.

Average Valuation Matrix and Representative Default Probabilities

Table 9 reports probabilities of default in a given year by income quintile for two representative states with low and high exemption levels: Alabama and Massachusetts. Table 10 reports averages across the final set of 36 states considered of their valuation matrix across income quintiles. Quantitatively, a household currently in the third income quintile values a dollar if he transitions to the fourth quintile next period at 63 cents, and he transitions to the second quintile at $1.77.

Table 9: Default Probabilities

	$\pi^{\mathcal{D}}$		$\pi^{m\|\mathcal{D}}$	
Quintile	Alabama	Massachusetts	Alabama	Massachusetts
First	0.0157	0.0068	0.6115	0.0934
Second	0.0228	0.0081	0.6359	0.1413
Third	0.0199	0.0060	0.5996	0.2433
Fourth	0.0115	0.0039	0.5933	0.3898
Fifth	0.0041	0.0007	0.6527	0.6817

Note: Table 9 reports two examples of yearly bankruptcy probabilities by income quintiles for the states of Alabama and Massachusetts, chosen as examples of states associated with high and low marginal welfare gains from increasing exemption levels. These values are constructed combining data on the universe of bankruptcies from the Department of Justice through the Integrated Database of the Federal Justice Center (ID-FJC) with total population data by state from the 2013 five-year ACS. The determination of income quintiles by state uses income data also from the 2013 five-year ACS.

Table 10: Average Valuation Matrix

	First	Second	Third	Fourth	Fifth
First	1	0.47	0.27	0.17	0.09
Second	2.64	1	0.59	0.38	0.20
Third	4.56	1.77	1	0.63	0.33
Fourth	7.35	2.93	1.65	1	0.53
Fifth	14.31	5.91	3.35	2.04	1

Note: Table 10 reports the average across the final set 36 states of their valuation matrix G. The determination of income quintiles by state uses income data from the 2013 five-year ACS. Households' risk aversion is $\gamma = 2$.

Marginal Welfare Changes Estimates

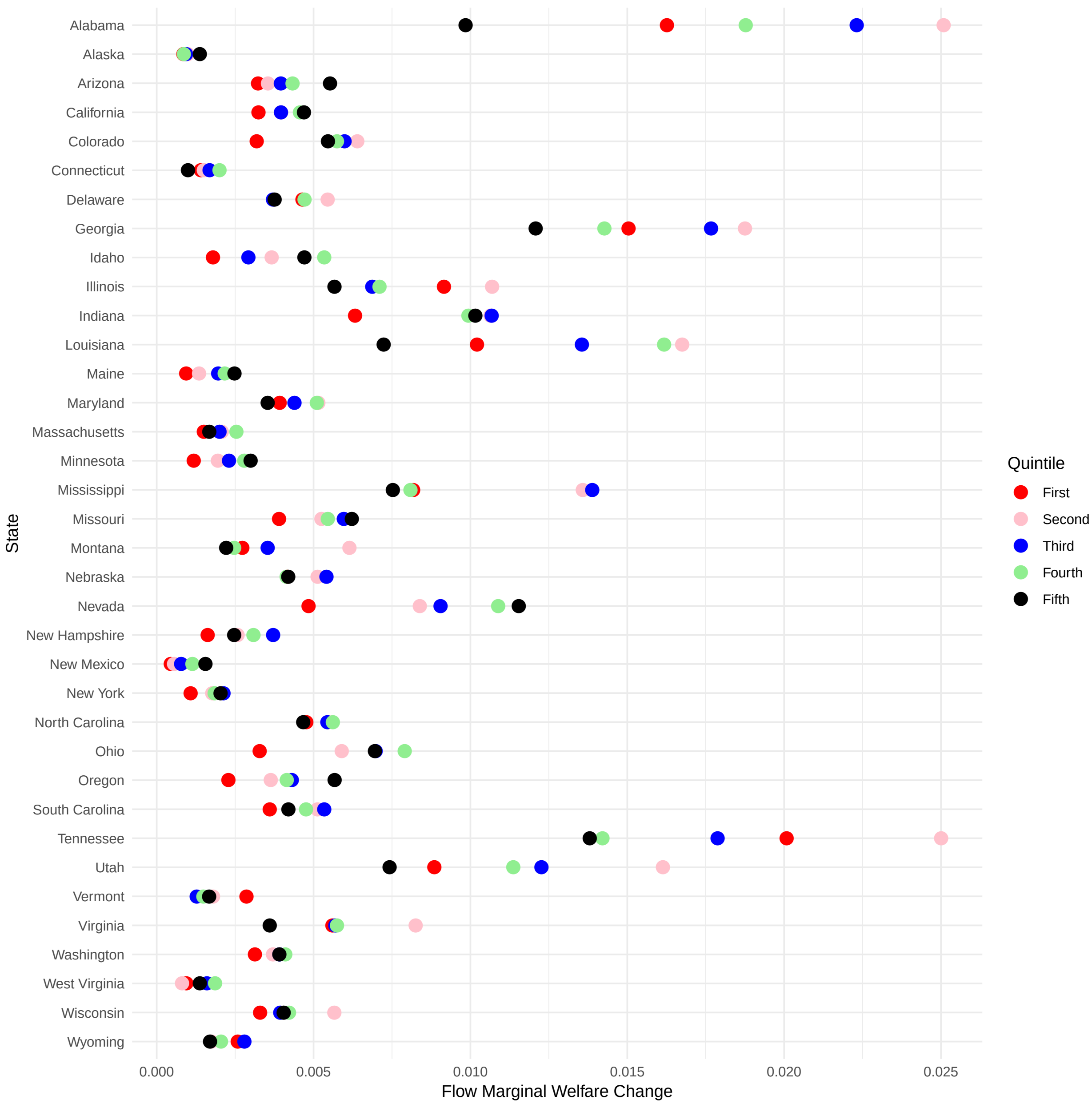

Figure 7: Flow Marginal Welfare Changes by Income Quintile

Note: Figure 7 shows the flow normalized marginal welfare changes by income quintiles by state, which corresponds to the vector F, defined in the statement of Proposition 3. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). First (fifth) quintile corresponds to the bottom (top) 20% of the income distribution. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$.

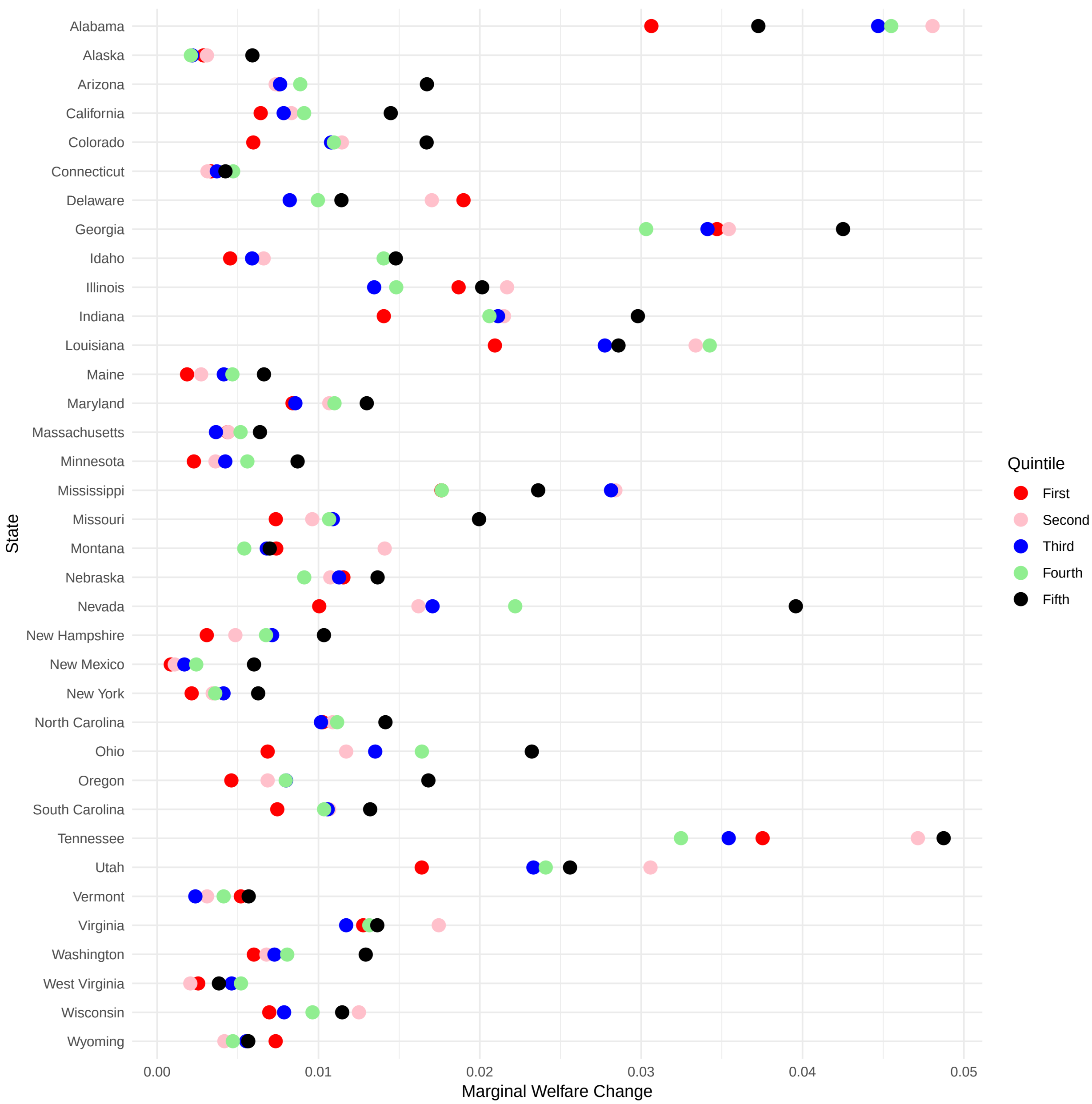

Figure 6: Marginal Welfare Changes by Income Quintile

Note: Figure 6 shows the normalized marginal welfare changes by income quintiles by state, calculated using Equation (23). The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). First (fifth) quintile corresponds to the bottom (top) 20% of the income distribution. Figure 4 in the text reports unweighted averages across quintiles by states of marginal welfare changes. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$.

D Proofs and Derivations

The Online Appendix provides a more detailed characterization of the equilibrium in Section 2, as well as additional proofs and derivations of results in the remaining sections.

Section 2

Proposition 1. (Directional test for a change in the exemption level m)

Proof. Given households' indirect utility for a given level of m, defined in Equation (6), and exploiting households' optimal default and borrowing choices, we can express $\frac{dW}{dm}$ as follows

$$\begin{aligned}\frac{dW}{dm} &= u'(c_0)\frac{\partial Q_0(b_1,m)}{\partial m} + \beta\int_{\tilde{s}}^{\hat{s}} u'\left(c_1^{\mathcal{D}}(s)\right)dF(s)\\ &+ \underbrace{\left[u'(c_0)\frac{\partial Q_0(b_1,m)}{\partial b_1} - \beta\int_{\hat{s}}^{\overline{s}} u'(n(s)-b_1)\,dF(s)\right]}_{=0}\frac{db_1}{dm}\\ &+ \beta\underbrace{\left[u(m)f(\hat{s})\frac{d\hat{s}}{dm} - u(m)f(\hat{s})\frac{d\hat{s}}{dm}\right]}_{=0},\end{aligned}$$

where $\tilde{s} = \max\{m, \underline{s}\}$. Therefore,

$$\frac{dW}{dm} = u'(c_0)\frac{\partial q_0}{\partial m}b_1 + \beta\int_{\tilde{s}}^{\hat{s}} u'\left(c_1^{\mathcal{D}}(s)\right)dF(s).$$

Dividing by $u'(c_0)$, we can express the normalized welfare change as follows

$$\frac{\frac{dW}{dm}}{u'(c_0)} = \frac{\partial Q_0(b_1,m)}{\partial m} + \beta\int_{\tilde{s}}^{\hat{s}}\frac{u'\left(c_1^{\mathcal{D}}(s)\right)}{u'(c_0)}dF(s) = \frac{\partial Q_0(b_1,m)}{\partial m} + \beta\int_{\tilde{s}}^{\hat{s}}dF(s)\,\frac{\int_{\tilde{s}}^{\hat{s}}\frac{u'\left(c_1^{\mathcal{D}}(s)\right)}{u'(c_0)}dF(s)}{\int_{\tilde{s}}^{\hat{s}}dF(s)},$$

which exactly corresponds to Equation (12) in the paper. □

Section 3

Proposition 2. (Directional test for a change in the exemption level m: general model)

Proof. I use the definitions of households' value functions when making the default/repayment decision, in Equation (15), and after repayment and defaulting, in Equations (16) and (17).[24] Exploiting the envelope theorem, the marginal welfare change in the value function of households i in period t after repaying and defaulting can be expressed as follows

$$\begin{aligned}\frac{dV_{i,t}^{\mathcal{N}}}{dm}(\Omega_i;m) &= \frac{\partial u_{i,t}(c_{i,t},h_{i,t};\Omega_i)}{\partial c_{i,t}}\sum_k\frac{\partial q_{i,t}^k(\Psi_i',\Omega_i,m)}{\partial m}b_{i,t+1}^k + \beta_i\frac{d\mathbb{E}_t\left[V_{i,t+1}(\Omega_i';m)\right]}{dm}\\ \frac{dV_{i,t}^{\mathcal{D}}}{dm}(\Omega_i;m) &= \frac{\partial u_{i,t}(c_{i,t},h_{i,t};\Omega_i)}{\partial c_{i,t}}\frac{\partial\mathcal{W}(m;\boldsymbol{a}_i,\boldsymbol{x}_i,\boldsymbol{s}_i)}{\partial m} + \beta_i\mathbb{E}_t\left[\frac{dV_{i,t+1}^{\mathcal{N}}(\Omega_i';m)}{dm}\right],\end{aligned}$$

[24] Equation 29 would feature an additional term if households' law of motion $F(\cdot)$ depended directly on i) the exemption level m or ii) other households' choices, that is, in general equilibrium.

where the expression for $\frac{d\mathbb{E}_t\left[V_{i,t+1}\left(\Omega_i';m\right)\right]}{dm}$ can be written as

$$\frac{d\mathbb{E}_t\left[V_{i,t+1}\left(\Omega_i';m\right)\right]}{dm} = \underbrace{\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\frac{dV_{i,t+1}^{\mathcal{D}}\left(\Omega_i';m\right)}{dm}\right]}_{\int_{\mathcal{D}_{t+1}}\frac{dV_{i,t+1}^{\mathcal{D}}\left(\Omega_i';m\right)}{dm}dF\left(\boldsymbol{s}_i'|\Omega_i\right)} + \underbrace{\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\frac{dV_{i,t+1}^{\mathcal{N}}\left(\Omega_i';m\right)}{dm}\right]}_{\int_{\mathcal{N}_{t+1}}\frac{dV_{i,t+1}^{\mathcal{N}}\left(\Omega_i';m\right)}{dm}dF\left(\boldsymbol{s}_i'|\Omega_i\right)}. \tag{29}$$

At any given period and state the partial derivative of the function $\mathcal{W}\left(\cdot\right)$ takes the form

$$\frac{\partial\mathcal{W}}{\partial m} = \begin{cases} 1, & \text{if } \mathcal{W}\left(\cdot\right) = m \\ 0, & \text{otherwise.} \end{cases}$$

Starting in period t, we can recursively substitute in the expression for $\frac{dV_{i,t}^{\mathcal{N}}}{dm}\left(\Omega_i;m\right)$, after suppressing the argument of most functions to simplify notation, to find that

$$\frac{dV_{i,t}^{\mathcal{N}}}{dm} = \frac{\partial u_{i,t}}{\partial c_{i,t}}\sum_k\frac{\partial q_{i,t}^k}{\partial m}b_{i,t+1}^k + \beta_i\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial\mathcal{W}_{t+1}}{\partial m} + \beta_i\mathbb{E}_{t+1}\left[\frac{dV_{i,t+2}^{\mathcal{N}}}{dm}\right]\right] + \beta_i\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\frac{dV_{i,t+1}^{\mathcal{N}}\left(\Omega_i';m\right)}{dm}\right]$$

where I denote by $\mathcal{W}_{t+1}$ the function $\mathcal{W}\left(\cdot\right)$ evaluated in period $t+1$. Further substitution implies that

$$\begin{aligned}\frac{dV_{i,t}^{\mathcal{N}}}{dm} = {} & \frac{\partial u_{i,t}}{\partial c_{i,t}}\sum_k\frac{\partial q_{i,t}^k}{\partial m}b_{i,t+1}^k + \beta_i\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\sum_k\frac{\partial q_{i,t+1}^k}{\partial m}b_{i,t+2}^k\right] + \beta_i\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial\mathcal{W}_{t+1}}{\partial m}\right] \\ & + \beta_i^2\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\mathbb{E}_{t+1}\left[\frac{dV_{i,t+2}^{\mathcal{N}}}{dm}\right]\right] + \beta_i^2\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\frac{d\mathbb{E}_{t+1}\left[V_{i,t+2}\right]}{dm}\right].\end{aligned}$$

Iterating forward one more period implies that

$$\begin{aligned}\frac{dV_{i,t}^{\mathcal{N}}}{dm} = {} & \frac{\partial u_{i,t}}{\partial c_{i,t}}\sum_k\frac{\partial q_{i,t}^k}{\partial m}b_{i,t+1}^k + \beta_i\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\sum_k\frac{\partial q_{i,t+1}^k}{\partial m}b_{i,t+2}^k\right] + \beta_i\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial\mathcal{W}_{t+1}}{\partial m}\right] \\ & + \beta_i^2\pi_{t,t+1}^{\mathcal{D}}\mathbb{E}_t^{\mathcal{D}}\left[\mathbb{E}_{t+1}\left[\frac{\partial u_{i,t+2}}{\partial c_{i,t+2}}\sum_k\frac{\partial q_{i,t+2}^k}{\partial m}b_{i,t+3}^k + \beta_i\frac{d\mathbb{E}_{t+2}\left[V_{i,t+3}\left(\Omega_i';m\right)\right]}{dm}\right]\right] \\ & + \beta_i^2\pi_{t,t+1}^{\mathcal{N}}\mathbb{E}_t^{\mathcal{N}}\left[\pi_{t+1,t+2}^{\mathcal{D}}\mathbb{E}_{t+1}^{\mathcal{D}}\left[\frac{dV_{i,t+2}^{\mathcal{D}}\left(\Omega_i';m\right)}{dm}\right] + \pi_{t+1,t+2}^{\mathcal{N}}\mathbb{E}_{t+1}^{\mathcal{N}}\left[\frac{dV_{i,t+2}^{\mathcal{N}}\left(\Omega_i';m\right)}{dm}\right]\right],\end{aligned}$$

and so on. Consequently, changing time indexes and starting in period τ, we can express the marginal welfare change for a household i that starts from a no-default state from the perspective of period τ as follows,

$$\frac{dW_{i,\tau}}{dm} = \sum_{t=\tau}^{T}\beta_i^{t-\tau}\pi_{\tau,t}^{\mathcal{N}}\mathbb{E}_\tau^{\mathcal{N}}\left[\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial\sum_k q_{i,t}^k b_{i,t+1}^k}{\partial m}\right] + \sum_{t=\tau}^{T}\beta_i^{t-\tau+1}\pi_{\tau,t+1}^{\mathcal{D}}\mathbb{E}_\tau^{\mathcal{D}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial\mathcal{W}_{t+1}}{\partial m}\right],$$

where $\pi_{\tau,t}^{\mathcal{N}}$ and $\pi_{\tau,t}^{\mathcal{D}}$ respectively denote equilibrium repayment and default probabilities for a given household in period t given period τ state variables, and $\mathbb{E}_{\tau,t}^{\mathcal{N}}\left[\cdot\right]$ and $\mathbb{E}_{\tau,t}^{\mathcal{D}}\left[\cdot\right]$ denote the conditional expectations on repayment and default states of period t random variables given period τ state variables. If households had access to credit markets in the same period in which they default, the second term in Equation (22) needs to be augmented and would read as $\sum_{t=\tau}^{T}\beta_i^{t-\tau+1}\pi_{\tau,t+1}^{\mathcal{D}}\mathbb{E}_\tau^{\mathcal{D}}\left[\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\left(\frac{\partial\mathcal{W}_{t+1}}{\partial m} + \frac{\partial\sum_k q_{i,t+1}^k b_{i,t+2}^k}{\partial m}\right)\right]$. $\square$

Section 4

Proposition 3. (Empirical Implementation)

Proof. Starting from Equation (18), given that households have homogeneous time-invariant preferences and that $T \to \infty$, it is possible to drop individual i and time t subscripts, to express $\frac{dV^{\mathcal{N}}}{dm}(\Omega)$ recursively as follows

$$\frac{dV^{\mathcal{N}}}{dm}(\Omega) = u'(c(\Omega)) \frac{\partial Q}{\partial m}(\Omega) + \beta \pi_{\Omega}^{\mathcal{D}} \mathbb{E}^{\mathcal{D}} \left[\frac{dV^{\mathcal{D}}(\Omega')}{dm} \middle| \Omega \right] + \beta \pi_{\Omega}^{\mathcal{N}} \mathbb{E}^{\mathcal{N}} \left[\frac{dV^{\mathcal{N}}(\Omega')}{dm} \middle| \Omega \right],$$

where the flow marginal cost term $\frac{\partial Q}{\partial m}(\Omega)$ is given by

$$\frac{\partial Q}{\partial m}(\Omega) = \sum_k \frac{\partial q^k}{\partial m}(\Omega) b^{k\prime}(\Omega).$$

Similarly, we can express $\frac{dV^{\mathcal{D}}}{dm}(\Omega')$ as follows

$$\frac{dV^{\mathcal{D}}}{dm}(\Omega') = u'(c(\Omega')) \frac{\partial \mathcal{W}}{\partial m}(\Omega') + \beta \int_{\mathcal{N}} \frac{dV^{\mathcal{N}}}{dm}(\Omega'') dF(\Omega''|\Omega').$$

After normalizing both expressions by the relevant state marginal utilities, we can rewrite them as

$$\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega))} = \frac{\partial Q}{\partial m}(\Omega) + \beta \int_{\mathcal{D}} \frac{u'(c(\Omega'))}{u'(c(\Omega))} \frac{\frac{dV^{\mathcal{D}}}{dm}(\Omega')}{u'(c(\Omega'))} dF(\Omega'|\Omega) + \beta \int_{\mathcal{N}} \frac{u'(c(\Omega'))}{u'(c(\Omega))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega')}{u'(c(\Omega'))} dF(\Omega'|\Omega)$$
$$\frac{\frac{dV^{\mathcal{D}}}{dm}(\Omega')}{u'(c(\Omega'))} = \frac{\partial \mathcal{W}}{\partial m}(\Omega') + \beta \int_{\mathcal{N}} \frac{u'(c(\Omega''))}{u'(c(\Omega'))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega'')}{u'(c(\Omega''))} dF(\Omega''|\Omega').$$

When combined, we find a single functional equation that exclusively depends on the values of $\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega))}$ for different states Ω, given by

$$\begin{aligned}\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega))} = {} & \frac{\partial Q}{\partial m}(\Omega) + \beta \int_{\mathcal{D}} \frac{u'(c(\Omega'))}{u'(c(\Omega))} \frac{\partial \mathcal{W}}{\partial m}(\Omega') dF(\Omega'|\Omega) \\ & + \beta \int_{\mathcal{N}} \frac{u'(c(\Omega'))}{u'(c(\Omega))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega')}{u'(c(\Omega'))} dF(\Omega'|\Omega) \\ & + \beta^2 \int_{\mathcal{D}} \frac{u'(c(\Omega'))}{u'(c(\Omega))} \left(\int_{\mathcal{N}} \frac{u'(c(\Omega''))}{u'(c(\Omega'))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega'')}{u'(c(\Omega''))} dF(\Omega''|\Omega') \right) dF(\Omega'|\Omega).\end{aligned}$$

In order to work with more intuitive expressions in matrix form, from now on I explicitly state whether a given variable (e.g., consumption) is in a default or no default state. This change to a more explicit notation allows us to express $\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega;\mathcal{N}))}$ as follows

$$\begin{aligned}\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega;\mathcal{N}))} = {} & \frac{\partial Q}{\partial m}(\Omega) + \beta \int_{\mathcal{D}} \frac{u'(c(\Omega';\mathcal{D}))}{u'(c(\Omega;\mathcal{N}))} \frac{\partial \mathcal{W}}{\partial m}(\Omega') dF(\Omega'|\Omega;\mathcal{N}) \\ & + \beta \int_{\mathcal{N}} \frac{u'(c(\Omega';\mathcal{N}))}{u'(c(\Omega;\mathcal{N}))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega')}{u'(c(\Omega';\mathcal{N}))} dF(\Omega'|\Omega;\mathcal{N}) \\ & + \beta^2 \int_{\mathcal{D}} \frac{u'(c(\Omega';\mathcal{D}))}{u'(c(\Omega;\mathcal{N}))} \left(\int_{\mathcal{N}} \frac{u'(c(\Omega'';\mathcal{N}))}{u'(c(\Omega';\mathcal{D}))} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega'')}{u'(c(\Omega'';\mathcal{N}))} dF(\Omega''|\Omega';\mathcal{D}) \right) dF(\Omega'|\Omega;\mathcal{N}).\end{aligned} \tag{30}$$

After discretizing the state space, we will be able to solve a system of equations that characterizes normalized welfare changes $\frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega)}{u'(c(\Omega;\mathcal{N}))}$ for different values of Ω. I now proceed to discretize the state space by assuming that the there is finite set of S possible states, denoted by $\Omega \in \left\{\Omega^1, \Omega^2, \ldots, \Omega^S\right\}$. The stacked vector of normalized marginal welfare changes for households that haven't defaulted for different state variables, denoted by $\frac{d\hat{W}}{dm}$ and given by

$$\frac{d\hat{W}}{dm} = V_m^{\mathcal{N}} = \left(\begin{array}{ccc} \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega^1)}{u'(c(\Omega^1;\mathcal{N}))}, & \ldots, & \frac{\frac{dV^{\mathcal{N}}}{dm}(\Omega^S)}{u'(c(\Omega^S;\mathcal{N}))} \end{array} \right)',$$

can be expressed as the solution to

$$V_m^{\mathcal{N}} = F + \beta \left(\Pi_{\mathcal{N}\to\mathcal{N}} \odot G_{\mathcal{N}\to\mathcal{N}}\right) \times V_m^{\mathcal{N}} + \beta^2 \left(\Pi_{\mathcal{N}\to\mathcal{D}} \odot G_{\mathcal{N}\to\mathcal{D}}\right) \times \left(\Pi_{\mathcal{D}\to\mathcal{N}} \odot G_{\mathcal{D}\to\mathcal{N}}\right) \times V_m^{\mathcal{N}}, \tag{31}$$

where $\times$ denotes standard matrix multiplication, $\odot$ denotes element-wise matrix multiplication (Hadamard product), and where X' for a matrix X denotes matrix transposition. I now describe each of the elements of Equation (31). The vector F is given by

$$F = Q_m + \beta \text{diag}\left(\Pi_{\mathcal{N}\to m} \times G'_{\mathcal{N}\to m}\right),$$

where each element of $Q_m = \left(\frac{\partial Q}{\partial m}\left(\Omega^1\right), \frac{\partial Q}{\partial m}\left(\Omega^2\right), \ldots, \frac{\partial Q}{\partial m}\left(\Omega^S\right)\right)'$, given by $\frac{\partial Q}{\partial m}\left(\Omega\right) = \sum_k \frac{\partial q^k}{\partial m}\left(\Omega\right) b^{k\prime}\left(\Omega\right)$, captures the marginal flow cost associated with higher interest rates. The matrix $\Pi_{\mathcal{N}\to m}$ corresponds to the transition matrix into bankruptcy claiming the full exemption, while $G_{\mathcal{N}\to m}$ corresponds to relative valuations towards those states, with each element given by $\frac{u'(c(\Omega'))}{u'(c(\Omega))}$. We can similarly define transition matrices between no-default ($\mathcal{N}$) and bankruptcy ($\mathcal{D}$), as those given by $\Pi_{\mathcal{N}\to\mathcal{N}}$, $\Pi_{\mathcal{N}\to\mathcal{D}}$, and $\Pi_{\mathcal{D}\to\mathcal{N}}$. In the same way, we can definite relative valuation matrices between these scenarios, as in $G_{\mathcal{N}\to m}$, $G_{\mathcal{N}\to\mathcal{N}}$, $G_{\mathcal{N}\to\mathcal{D}}$, and $G_{\mathcal{D}\to\mathcal{N}}$. I provide explicit characterizations of these objects for the specific application implemented in the paper in the Online Appendix.

The solution to the system for $V_m^{\mathcal{N}}$ defined in Equation (31) is therefore given by

$$\frac{d\hat{W}}{dm} = V_m^{\mathcal{N}} = A^{-1}F,$$

where F is described above and A is given by

$$A = \mathbb{I} - \beta\left(\Pi_{\mathcal{N}\to\mathcal{N}} \odot G_{\mathcal{N}\to\mathcal{N}}\right) - \beta^2 \left(\Pi_{\mathcal{N}\to\mathcal{D}} \odot G_{\mathcal{N}\to\mathcal{D}}\right) \times \left(\Pi_{\mathcal{D}\to\mathcal{N}} \odot G_{\mathcal{D}\to\mathcal{N}}\right),$$

which corresponds to Equation (23) in the text, and concludes the proof. □

Online Appendix

E Proofs and Derivations

Section 2

To simplify the exposition, and without loss of generality, I assume that $n(s) = s$, which simplifies the characterization of default thresholds. I also assume that $\underline{s} > 0$ and that $m \in [0, \overline{s}]$, which guarantees that the bankruptcy procedure does not rely on external funds.

Borrowers' default decision In general, households' default decision can be characterized by a threshold $\hat{s}$, given by

$$\hat{s} \equiv \max\{\underline{s}, \min\{\overline{s}, m + b_1\}\},$$

such that households default when $s < \hat{s}$ and repay otherwise.[25] We can express the derivatives of $\hat{s}$ with respect to m and b_1 wherever $\hat{s}$ is differentiable as follows

$$\frac{\partial \hat{s}}{\partial m} = \frac{\partial \hat{s}}{\partial b_1} = \mathbb{I}[\underline{s} < m + b_1 < \overline{s}],$$

where $\mathbb{I}[\cdot]$ denotes the indicator function.

Credit supply schedule Throughout the Appendix, to simplify many characterizations, I use the credit supply schedule $Q_0(b_1, m)$, which denotes the total amount raised by households at date 0 when they take on debt with face value b_1 at date 1. This schedule can be written as the product of the face value of debt, b_1, and the price of debt, $q_0(b_1, m)$, as follows

$$Q_0(b_1, m) = q_0(b_1, m)\, b_1,$$

where $\frac{\partial Q_0}{\partial b_1} = q_0 + \frac{\partial q_0}{\partial b_1} b_1$ and $\frac{\partial Q_0}{\partial m} = \frac{\partial q_0}{\partial m} b_1 = \frac{\partial \log q_0}{\partial m} q_0 b_1 = -\frac{\partial \log(1+r_0)}{\partial m} q_0 b_1$.

Given households' optimal default decision, we can express the equilibrium credit supply schedule offered by lenders as follows

$$Q_0(b_1, m) \equiv q_0(b_1, m)\, b_1 = \begin{cases} \frac{\delta \int_{\underline{s}}^{\hat{s}} \max\{n(s)-m,0\} dF(s) + b_1 \int_{\hat{s}}^{\overline{s}} dF(s)}{1+r^{\ell}}, & b_1 > 0 \\ \frac{b_1}{1+r^{\ell}} & b_1 \leq 0, \end{cases}$$

Note that $Q_0(0, m) = 0$ and that $\lim_{b_1 \to 0^+} Q_0(b_1, m) = \lim_{b_1 \to 0^-} Q_0(b_1, m) = 0$, which implies that $Q_0(b_1, m)$ is continuous at $b_1 = 0$. Note also that

$$\lim_{b_1 \to \infty} Q_0(b_1, m) = \frac{\delta \int_{\underline{s}}^{\overline{s}} \max\{n(s) - m, 0\} dF(s)}{1 + r^{\ell}},$$

which corresponds to lenders receiving all non-exempt resources after accounting for deadweight losses.

Two properties of the debt pricing schedule are important for the analysis. First, the pricing schedule decreases (interest rates increase) with the level of debt b_1. Second, the pricing schedule decreases (interest

[25] The unconditional probability of filing for bankruptcy while claiming the full exemptions can be expressed as the product of the unconditional probability of filing for bankruptcy times the probability of claiming the full exemption conditional on filing for bankruptcy. Formally, $\pi_m = \pi_{\mathcal{D}} \pi_{m|\mathcal{D}}$, since $\int_{\tilde{s}}^{\hat{s}} dF(s) = \int_{\underline{s}}^{\hat{s}} dF(s) \frac{\int_{\tilde{s}}^{\hat{s}} dF(s)}{\int_{\underline{s}}^{\hat{s}} dF(s)}$.

rates increase) with the level of the bankruptcy exemption. Formally, $\frac{\partial q_0(b_1,m)}{\partial b_1} < 0$ and $\frac{\partial q_0(b_1,m)}{\partial m} < 0$. For a given level of m, the required interest rate spread increases with the amount of credit issued. This occurs for two reasons. First, the per unit fraction of liabilities recovered by lenders in default states decreases with the total amount of credit. Second, because the default region widens, more resources are lost as bankruptcy costs. Also, for a given level of b_1, the required interest rate increases with the level of the bankruptcy exemption. There are again two reasons for this. First, the recovery rate for lenders in default states decreases with the level of the exemption, since households get to keep a higher exemption. Second, because the default region widens, more resources are lost as bankruptcy costs.

Whenever $b_1 > 0$, we can find that

$$\frac{\partial Q_0(b_1,m)}{\partial b_1} = \frac{\int_{\hat{s}}^{\overline{s}} dF(s) - (1-\delta) f(\hat{s}) b_1 \frac{\partial \hat{s}}{\partial b_1}}{1+r^{\ell}} \gtrless 0$$
$$\frac{\partial Q_0(b_1,m)}{\partial m} = \frac{\delta \int_{\underline{s}}^{\hat{s}} \frac{\partial \max\{n(s)-m,0\}}{\partial m} dF(s) - (1-\delta) f(\hat{s}) b_1 \frac{\partial \hat{s}}{\partial m}}{1+r^{\ell}} \leq 0,$$

where we can write $\frac{\partial \max\{n(s)-m,0\}}{\partial m} = -\mathbb{I}[n(s) > m]$.[26] While $\frac{\partial Q_0(b_1,m)}{\partial b_1}$ can be positive or negative, it is always the case that $\frac{\partial Q_0(b_1,m)}{\partial m}$ is negative. Note that $Q_0(b_1,m)$ can be non-differentiable at $b_1 = 0$, since

$$\lim_{b_1 \to 0^+} \frac{\partial Q_0(b_1,m)}{\partial b_1} = \frac{\int_{\hat{s}}^{\overline{s}} dF(s)}{1+r^{\ell}} > 0$$
$$\lim_{b_1 \to 0^-} \frac{\partial Q_0(b_1,m)}{\partial b_1} = \frac{1}{1+r^{\ell}} > 0.$$

It follows immediately that the right limit will be less than $\frac{1}{1+r^{\ell}}$ whenever $\underline{s} < m$.

Note also that $Q_0(b_1,m)$ will be non-differentiable when $b_1 = \overline{s} - m$, since

$$\lim_{b_1 \to (\overline{s}-m)^+} \frac{\partial Q_0(b_1,m)}{\partial b_1} = 0$$
$$\lim_{b_1 \to (\overline{s}-m)^-} \frac{\partial Q_0(b_1,m)}{\partial b_1} = -(1-\delta) f(\overline{s})(\overline{s}-m) \leq 0.$$

The fact that the left limit is negative, combined with the fact that $\lim_{b_1 \to 0^+} \frac{\partial Q_0(b_1,m)}{\partial b_1} > 0$ and the continuity of $Q_0(b_1,m)$, implies that $Q_0(b_1,m)$ must feature at least an interior maximum whenever $\delta < 1$ (and $m < \overline{s}$). Figure 10 illustrates these analytical results in the context of the numerical simulation studied in Section 2.4.

Finally, the curvature of $Q_0(b_1,m)$ for the region $\underline{s} < m + b_1 < \overline{s}$, can be expressed as

$$\frac{\partial^2 Q_0(b_1,m)}{\partial b_1^2} = \frac{-f(\hat{s}) - (1-\delta)(f'(\hat{s}) b_1 + f(\hat{s}))}{1+r^{\ell}}.$$

Therefore, it is possible to guarantee that $\frac{\partial^2 Q_0}{\partial b_1^2} > 0$ by imposing a mild regularity condition that prevents the $f(s)$ from varying too quickly, given by

$$\frac{d \log f(s)}{d \log(s)} = \hat{s} \frac{f'(\hat{s})}{f(\hat{s})} > -1. \tag{32}$$

[26] The response of equilibrium interest rates – not credit supply schedules – to changes in the exemption level is given by

$$\frac{dq_0(b_1,m)}{dm} = \frac{\partial q_0(b_1,m)}{\partial m} + \frac{\partial q_0(b_1,m)}{\partial b_1} \frac{db_1}{dm},$$

which is negative when $\frac{db_1}{dm} > 0$, but can in principle take any sign. As discussed in Section 4, the empirical approach is designed to recover the counterpart of $\frac{\partial q_0(b_1,m)}{\partial m}$.

We can also express the cross partial $\frac{\partial^2 Q_0}{\partial b_1 \partial m}$ as follows

$$\frac{\partial^2 Q_0}{\partial b_1 \partial m} = \frac{-f(\hat{s}) \frac{\partial \hat{s}}{\partial m} - (1-\delta) f'(\hat{s}) b_1 \frac{\partial \hat{s}}{\partial b_1} \frac{\partial \hat{s}}{\partial m}}{1+r^{\ell}} \leq 0,$$

which implies that the price obtained per unit of debt by households is lower at the margin when exemption levels are higher.

Borrowers' debt choice We define the households' objective function by $J(b_1, m)$ as follows

$$J(b_1, m) = u(c_0) + \beta \tilde{V}(b_1, m),$$

where $c_0 = n_0 + Q_0(b_1, m)$, and where $\tilde{V}(b_1, m)$ is corresponds to

$$\begin{aligned}\tilde{V}(b_1, m) &= \int_{\underline{s}}^{\hat{s}} u\left(c_1^{\mathcal{D}}(s)\right) dF(s) + \int_{\hat{s}}^{\overline{s}} u\left(c_1^{\mathcal{N}}(s)\right) dF(s) \\ &= \int_{\underline{s}}^{\hat{s}} u(\min\{n(s), m\}) dF(s) + \int_{\hat{s}}^{\overline{s}} u(n(s) - b_1) dF(s).\end{aligned}$$

It should be evident that the households' objective function $J(b_1, m)$ inherits the continuity and differentiability properties of $Q_0(b_1, m)$. Whenever it is differentiable, we can express households' first-order condition for b_1 as follows

$$\frac{dJ}{db_1} = u'(c_0) \frac{\partial Q}{\partial b_1} - \beta \int_{\hat{s}}^{\overline{s}} u'(n(s) - b_1) dF(s) = 0. \tag{33}$$

Consequently, at any interior optimum, it must be the case that $\frac{\partial Q}{\partial b_1} > 0$, since the second term of Equation (33) is always negative.

Note that

$$\begin{aligned}\lim_{b_1 \to 0^+} \frac{dJ}{db_1} &= u'(n_0) \frac{\int_{\hat{s}}^{\overline{s}} dF(s)}{1+r^{\ell}} - \beta \int_{\hat{s}}^{\overline{s}} u'(n(s)) dF(s) \\ \lim_{b_1 \to (\overline{s}-m)^-} \frac{dJ}{db_1} &= -u'(n_0) \frac{(1-\delta) f(\overline{s})(\overline{s}-m)}{1+r^{\ell}} < 0.\end{aligned} \tag{34}$$

Provided that $m < \overline{s}$, there are primitives (n_0, $F(s)$, β, and r^{ℓ}) such that households find optimal to borrow, that is, to set $b_1 > 0$. This occurs when the initial endowment n_0 is sufficiently low, when households are very impatient, $\beta \to 0$, or when the level of expected future income is sufficiently large and not too stochastic (to prevent the precautionary savings effect from dominating).

The convexity of $J(b_1, m)$ is determined by

$$\frac{d^2 J}{db_1^2} = u''(c_0) \left(\frac{\partial Q}{\partial b_1}\right)^2 + u'(c_0) \frac{\partial^2 Q}{\partial b_1^2} + \beta \int_{\hat{s}}^{\overline{s}} u''(n(s) - b_1) dF(s) + \beta u'(\hat{s}) f(\hat{s}) \frac{\partial \hat{s}}{\partial b_1}.$$

The first and third terms of this expression are always negative and the second term will be negative provided that the regularity condition in Equation (32) holds. For standard choices of $u(c)$ and $F(s)$, the last term is never sufficiently strong to dominate the others, so the borrower's problem typically features a unique interior solution. As shown in Figure 11, the marginal benefit and cost of borrowing may be decreasing in b_1.

It is possible to find how households' debt choices vary with the level of m_1 in equilibrium. Formally,

$$\frac{db_1}{dm} = \frac{\overbrace{u''(c_0)\frac{\partial Q_0}{\partial m}\frac{\partial Q_0}{\partial b_1}}^{\text{Income effect }(>0)} + \overbrace{u'(c_0)\frac{\partial^2 Q_0}{\partial b_1 \partial m}}^{\text{Substitution effect }(<0)} \overbrace{+\beta u'(m) f(\hat{s})\frac{\partial \hat{s}}{\partial m}}^{\text{Direct effect }(>0)}}{-\frac{d^2 J}{db_1^2}}. \tag{35}$$

The first two effects, income and substitution, operate by varying the marginal benefit of borrowing. First, an increase in m reduces the amount raised by $\frac{\partial Q_0}{\partial m} < 0$, which reduces households' date 0 consumption c_0 and increases date 0 marginal utility $u'(c_0)$. This is an income effect that makes households more willing to borrow to increase their date 0 consumption. Second, under a mild regularity condition on $F(s)$, described in the Appendix, an increase in m reduces the marginal amount raised at date 0 by $\frac{\partial^2 Q_0}{\partial b_1 \partial m} < 0$. This is a substitution effect that makes borrowing less appealing at the margin, inducing households to borrow less. The third effect operates by varying the marginal cost of borrowing. All else equal, an increase in m widens the default region, which reduces the likelihood of having to repay the debt, making borrowing more attractive. The direct effect captures the notion that households borrow more ex-ante when exemptions are higher, since they anticipate having to repay their debt in fewer scenarios: this effect is often described as moral hazard.

Similar ambiguous comparative statistics can be derived for default probabilities, which depend on $\hat{s}$ which is a function b_1. Quintin (2012) shows in a similar class of models an ambiguous relation between leniency of bankruptcy systems and default rates.

Comparative statistics in credit markets are often studied by analyzing shifts in credit supply and credit demand. In the environment consider here, in which households understand that their borrowing choices affect the interest rate that they receive, there is not a well-defined credit demand curve, since households behave like monopolists. Nonetheless, it should be clear that the income and substitution effects identified in Equation (35) are responses to changes in the price of credit, which can be interpreted as credit supply effects. The direct/moral hazard effect is instead a pure shift in the desire of households to borrow that emerges from having to repay debts less often, so it can be interpreted as a credit demand effect.

As in most normative exercises, it is hard to guarantee the convexity of $W(m)$ in general, although numerical solutions with isoelastic preferences and log-normal uncertainty are remarkably well-behaved. Formally, we can express $\frac{d^2 W}{dm^2}$ as follows

$$\frac{d^2 W}{dm^2} = \underbrace{u''(c_0)\frac{dQ_0}{dm}\frac{\partial Q_0}{\partial m} + u'(c_0)\frac{d\left(\frac{\partial Q_0}{\partial m}\right)}{dm}}_{\frac{d\left(u'(c_0)\frac{\partial Q_0}{\partial m}\right)}{dm}} + \underbrace{\beta\left(\int_{\tilde{s}}^{\hat{s}} u''\left(c_1^{\mathcal{D}}\right) dF(s) + u'(\hat{s}) f(\hat{s})\frac{d\hat{s}}{dm} - u'(m) f(m)\frac{d\tilde{s}}{dm}\right)}_{\frac{d\left(\int_{\tilde{s}}^{\hat{s}} u'\left(c_1^{\mathcal{D}}\right) dF(s)\right)}{dm}},$$

which can take positive and negative values, as illustrated in Figure 12. In the baseline model, because the first-best outcome involves risk neutral lenders providing a flat consumption profile (full insurance) to risk averse borrowers at date 1, an exemption level that does not vary with the state s is optimal.

Section 4

Construction of Π matrices Given the available data and the use of income quintiles as the single state variables, I construct measures for the set of matrices Π as follows:

$$
\begin{aligned}
\Pi_{\mathcal{N}\to\mathcal{N}} &= \Pi_{\Omega} \odot \left(\mathbf{1}_S \times \pi'_{\mathcal{N}|\Omega}\right) \\
\Pi_{\mathcal{N}\to\mathcal{D}} &= \Pi_{\Omega} \odot \left(\mathbf{1}_S \times \pi'_{\mathcal{D}|\Omega}\right) \\
\Pi_{\mathcal{N}\to m} &= \Pi_{\Omega} \odot \left(\mathbf{1}_S \times \left(\boldsymbol{\pi}_{\mathcal{D}|\Omega} \odot \boldsymbol{\pi}_{m|\mathcal{D},\Omega}\right)'\right) \\
\Pi_{\mathcal{D}\to\mathcal{N}} &= \Pi_{\Omega},
\end{aligned}
\tag{36}
$$

where denotes the unconditional transition matrix between income quintiles, $\mathbf{1}_S$ denotes a vector of ones of dimension S, and the vectors $\pi_{\mathcal{N}|\Omega}$, $\pi_{\mathcal{D}|\Omega}$, and $\pi_{m|\mathcal{D},\Omega}$ are given by

$$
\boldsymbol{\pi}_{\mathcal{N}|\Omega} = \begin{pmatrix} \pi_{\mathcal{D}|\Omega^1} \\ \vdots \\ \pi_{\mathcal{D}|\Omega^S} \end{pmatrix}, \quad \boldsymbol{\pi}_{\mathcal{D}|\Omega} = \begin{pmatrix} \pi_{\mathcal{D}|\Omega^1} \\ \vdots \\ \pi_{\mathcal{D}|\Omega^S} \end{pmatrix}, \quad \boldsymbol{\pi}_{m|D,\Omega} = \begin{pmatrix} \pi_{m|\mathcal{D},\Omega^1} \\ \vdots \\ \pi_{m|\mathcal{D},\Omega^S} \end{pmatrix},
$$

while the matrix Π_{Ω} corresponds to

$$
\Pi_{\Omega} = \begin{pmatrix} P\left(\Omega^1|\Omega^1\right) & \dots & P\left(\Omega^S|\Omega^1\right) \\ \vdots & \ddots & \vdots \\ P\left(\Omega^1|\Omega^S\right) & \dots & P\left(\Omega^S|\Omega^S\right) \end{pmatrix}.
$$

In the empirical implementation in the paper, Ω^1 through Ω^S simply correspond to income quintiles. More generally, one could estimate directly all the transition matrices, or simply estimates two versions of the matrix Π_{Ω} depending on whether households start from a default or no default state. Note that in most models, given the set of states households default deterministically, but by introducing some randomness between the realization of the state variable the default decision, it is possible to make default a random outcome for a given set of state variables.

Tri-annual Calibration It is necessary to transform the different sources into consistent tri-annual estimates. First, because the PSID data is gather bi-annually, it is necessary to transform the estimated transition as follows $\Pi_{\Omega} = \left(\Pi_{\Omega}^{bi}\right)^{3/2}$, where Π_{Ω}^{bi} denotes the biannually estimated transition matrix. Second, since the sensitivity of credit supply to exemptions is estimated using annual rates, the marginal cost of each source of credit should be written as $-3\frac{\partial \log(1+r)}{\partial m} q_0^3 b_1$, where $\frac{\partial \log(1+r)}{\partial m}$ is the annual elasticity estimated from Regression R1. Third, the probability of declaring bankruptcy conditional on reaching a given income state also needs to be adjusted. We can express the probability of failure over three years (under the assumption that yearly probabilities are independent) as follows

$$
\begin{aligned}
\pi_{\mathcal{D}|\Omega}^{tri} &= \pi_{\mathcal{D}|\Omega}^{ann} + \pi_{\mathcal{D}|\Omega}^{ann}\left(1 - \pi_{\mathcal{D}|\Omega}^{ann}\right) + \pi_{\mathcal{D}|\Omega}^{ann}\left(1 - \pi_{\mathcal{D}|\Omega}^{ann}\right)^2, \\
&= \pi_{\mathcal{D}|\Omega}^{ann}\left[1 + \left(1 - \pi_{\mathcal{D}|\Omega}^{ann}\right) + \left(1 - \pi_{\mathcal{D}|\Omega}^{ann}\right)^2\right],
\end{aligned}
$$

which can be approximated as $\pi_{\mathcal{D}|\Omega}^{tri} \approx 3\pi_{\mathcal{D}|\Omega}^{ann}$ when $\pi_{\mathcal{D}|\Omega}^{ann}$ is small. Finally, the calibration of the discount factor $\beta = (0.94)^3$ already accounts for the tri-annual nature of the calibration, while the matrices of relative valuations do not need to be modified.

Section 5

Belief Distortions In order to account for the possibility that households make borrowing choices subject to distorted beliefs, I model households as borrowing according to the following objective function

$$\max u(c_0) + \beta \tilde{\mathbb{E}}\left[u(c_1(s))\right],$$

which can be expressed, as in the characterization of the baseline model, as follows,

$$J(b_1, m) = u(c_0) + \beta \int_{\underline{s}}^{\hat{s}} u\left(c_1^{\mathcal{D}}(s)\right) d\tilde{F}(s) + \beta \int_{\hat{s}}^{\overline{s}} u\left(c_1^{\mathcal{N}}(s)\right) d\tilde{F}(s),$$

where $c_0 = n_0 + Q_0(b_1, m)$. As in the baseline model, it is possible to characterize households' borrowing decisions through a first-order condition

$$u'(c_0) \frac{\partial Q_0}{\partial b_1} = \beta \int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) d\tilde{F}(s).$$

The change in social welfare, defined in the text to account for the difference in probability assessments between households and the planner, induced by a change in m, corresponds to

$$\begin{aligned}\frac{dW}{dm} = {} & u'(c_0) \frac{\partial Q_0(b_1, m)}{\partial m} + \beta \int_{m}^{\hat{s}} u'\left(c_1^{\mathcal{D}}\right) dF(s) \\ & + \left[u'(c_0) \frac{\partial Q_0(b_1, m)}{\partial b_1} - \beta \int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s)\right] \frac{db_1}{dm} \\ & + \underbrace{\beta\left[u(m) f(\hat{s}) \frac{d\hat{s}}{dm} - u(m) f(\hat{s}) \frac{d\hat{s}}{dm}\right]}_{=0},\end{aligned}$$

where the term in brackets can be rewritten by substituting households' optimality condition to yield Equation (26) in the text.

Lenders' Market Power The derivation of Equation (27) is identical to the derivation of Proposition 1. It is possible to define a credit supply with positive profits as follows

$$Q_0(b_1, m) = \max\left\{\frac{\delta \int_{\mathcal{D}} \max\{n(s) - m, 0\} dF(s) + b_1 \int_{\mathcal{N}} dF(s)}{1 + r^{\ell}} - \overline{\Pi}, 0\right\},$$

where $\overline{\Pi} \geq 0$ denotes the level of profits made by lenders in equilibrium.

General Equilibrium Externalities When households are initially endowed by a_0 units of an asset that trades at prices p_0 and $p_1(s)$, households' maximize

$$\max u(c_0) + \beta \mathbb{E}\left[V(a_1, b_1, s; m)\right],$$

where $V(a_1, b_1, s; m)$ denotes the continuation value of a household at the beginning of date 1, as a function of his asset position, a_1, debt position , b_1, realization of the state, s, and the exemption level m,

$$V(a_1, b_1, s; m) = \max\left\{u\left(c_1^{\mathcal{D}}(s)\right), u\left(c_1^{\mathcal{N}}(s)\right)\right\}.$$

Households' budget constraint at date 0 is given by

$$p_0 a_1 + c_0 = n_0 + p_0 a_0 + Q_0(b_1, m),$$

while date 1 budget constraints are given by

$$c_1^{\mathcal{N}}(s) = n_1(s) + p_1(s)\, a_1 - b_1$$
$$c_1^{\mathcal{D}}(s) = \min\{n_1(s) + p_1(s)\, a_1, m\}.$$

Households' optimality conditions for b_1 and a_1 are respectively given by

$$u'(c_0)\frac{\partial Q_0}{\partial b_1} = \beta \int_{\mathcal{N}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s)$$
$$u'(c_0)\, p_0 = \int_{\mathcal{D}_y} u'\left(c_1^{\mathcal{D}}(s)\right) p_1(s)\, dF(s) + \int_{\mathcal{N}} u'\left(c_1^{\mathcal{N}}(s)\right) p_1(s)\, dF(s),$$

where $\mathcal{D}_y$ denotes the region in which households default but do not claim the full exemptions. The change in social welfare induced by a change in m, corresponds to

$$\frac{dW}{dm} = u'(c_0)\frac{\partial q_0}{\partial m} b_1 + \beta \int_{\mathcal{D}_m} u'\left(c_1^{\mathcal{D}}(s)\right) dF(s)$$
$$+ u'(c_0)\frac{dp_0}{dm}\Delta a_1 + \beta \int_{\mathcal{D}_y} u'\left(c_1^{\mathcal{D}}(s)\right) \frac{dp_1}{dm} a_1 dF(s) + \beta \int_{\mathcal{N}} u'\left(c_1^{\mathcal{N}}(s)\right) \frac{dp_1}{dm} a_1 dF(s),$$

as shown in the text.

F Additional Material

This section includes exemption levels in 2008 and 2016, as well as three tables reporting a sensitivity analysis on the main empirical results that varies the value of credit supply sensitivities and households' risk aversion.

Exemption Levels

Figure 8: Evolution of Exemption Levels

Note: Figure 8 shows the level of total exemption levels (calculated as the sum of homestead and personal exemptions for a married couple filing jointly) in 2008 and 2016, measured in current dollars of the respective year. For the purpose of this plot, I set the homestead exemption of states with unlimited exemptions to $999,999.

Sensitivity Analysis

Table 11: Sensitivity Analysis (High Credit Supply Sensitivities)

	$\frac{d\hat{W}}{dm}$ (Marginal Welfare Change)					F (Flow Change)				
Quintile	Mean	S.D.	Pctl(10)	Median	Pctl(90)	Mean	S.D.	Pctl(10)	Median	Pctl(90)
First	0.0095	0.0091	0.0019	0.0065	0.0195	0.0044	0.0045	0.0009	0.0030	0.0095
Second	0.0127	0.0124	0.0020	0.0097	0.0310	0.0063	0.0065	0.0009	0.0048	0.0160
Third	0.0110	0.0105	0.0025	0.0069	0.0272	0.0056	0.0053	0.0011	0.0037	0.0134
Fourth	0.0113	0.0101	0.0024	0.0080	0.0258	0.0052	0.0045	0.0012	0.0038	0.0121
Fifth	0.0142	0.0114	0.0030	0.0109	0.0306	0.0041	0.0032	0.0009	0.0034	0.0089
Overall	0.0117	0.0107	0.0022	0.0086	0.0288	0.0051	0.0049	0.0009	0.0036	0.0129

Note: Table 11 reports summary statistics of the distribution of normalized marginal welfare changes by state associated with a one-dollar increase in the exemption level. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income levels, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$. This table uses credit supply sensitivities one higher standard deviation larger than the estimates from columns 2 and 4 in Table 2. That is, the sensitivity for unsecured credit is $0.0723 = 0.0185 + 0.0538$, while the sensitivity for auto loans is $0.0441 = 0.0240 + 0.0201$.

Table 12: Sensitivity Analysis ($\gamma = 0$)

	$\frac{d\hat{W}}{dm}$ (Marginal Welfare Change)					F (Flow Change)				
Quintile	Mean	S.D.	Pctl(10)	Median	Pctl(90)	Mean	S.D.	Pctl(10)	Median	Pctl(90)
First	0.0136	0.0119	0.0033	0.0099	0.0261	0.0058	0.0056	0.0015	0.0041	0.0119
Second	0.0126	0.0110	0.0030	0.0092	0.0299	0.0065	0.0060	0.0015	0.0048	0.0156
Third	0.0104	0.0076	0.0032	0.0081	0.0214	0.0055	0.0043	0.0015	0.0041	0.0120
Fourth	0.0100	0.0059	0.0038	0.0085	0.0188	0.0049	0.0031	0.0017	0.0042	0.0095
Fifth	0.0107	0.0064	0.0045	0.0088	0.0186	0.0038	0.0022	0.0014	0.0034	0.0067
Overall	0.0114	0.0089	0.0034	0.0088	0.0222	0.0053	0.0045	0.0014	0.0040	0.0108

Note: Table 12 reports summary statistics of the distribution of normalized marginal welfare changes by state associated with a one-dollar increase in the exemption level. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income levels, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 0$.

Table 13: Sensitivity Analysis ($\gamma = 4$)

	$\frac{d\hat{W}}{dm}$ (Marginal Welfare Change)					F (Flow Change)				
Quintile	Mean	S.D.	Pctl(10)	Median	Pctl(90)	Mean	S.D.	Pctl(10)	Median	Pctl(90)
First	0.0091	0.0081	0.0021	0.0064	0.0187	0.0043	0.0041	0.0009	0.0031	0.0090
Second	0.0219	0.0230	0.0033	0.0125	0.0576	0.0092	0.0094	0.0015	0.0062	0.0234
Third	0.0214	0.0236	0.0042	0.0116	0.0497	0.0090	0.0096	0.0022	0.0053	0.0207
Fourth	0.0283	0.0405	0.0048	0.0153	0.0573	0.0101	0.0129	0.0021	0.0059	0.0208
Fifth	0.0524	0.0586	0.0098	0.0345	0.0964	0.0105	0.0111	0.0020	0.0084	0.0176
Overall	0.0266	0.0377	0.0034	0.0136	0.0590	0.0086	0.0100	0.0015	0.0053	0.0191

Note: Table 13 reports summary statistics of the distribution of normalized marginal welfare changes by state associated with a one-dollar increase in the exemption level. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income levels, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 4$.

G Numerical Simulation

Figure 9 shows the cdf and pdf of the distribution of date 1 states or, equivalently, given that $n(s) = s$, the distribution of date 1 endowment realizations.

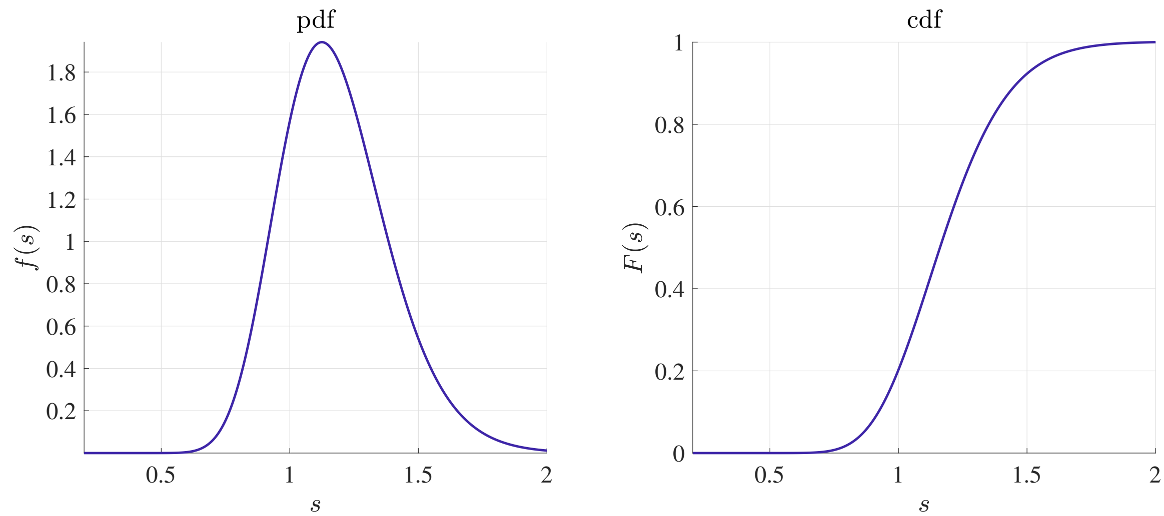

Figure 9: Date 1 Endowment Distribution $F(s)$

Note: Figure 9 shows the pdf and cdf of the date 1 endowment, since $n(s) = s$. The distribution $F(s)$ corresponds to a log-normal distribution with mean $\mu = 0.15$ and standard deviation $\sigma = 0.18$ of the underlying normal distribution, truncated to be in the interval $[\underline{s}, \overline{s}] = [0.2, 2]$. The mean of the distribution is 1.18 and its standard deviation is 0.21.

Figure 10 shows how the credit supply schedule varies with b_1 and m. The left plots fix the exemption level to $m = 0.7$ and vary the amount of borrowing b_1. The right plots fix the amount of borrowing to $b_1 = 0.4$ and vary the exemptions level. From top to bottom, each row respectively shows i) total amount raised $Q_0(b_1, m)$, ii) unit amount raised $q_0(b_1, m)$, iii) default and no default probabilities, $\pi_{\mathcal{D}}$ and $\pi_{\mathcal{N}}$.

Figure 11 illustrates the problem solved by households for a given exemption level $m = 0.7$. The top left plot shows households objective function for a given choice of b_1. The bottom left plot shows the amount of credit that a household raises for each promised repayment b_1. The three

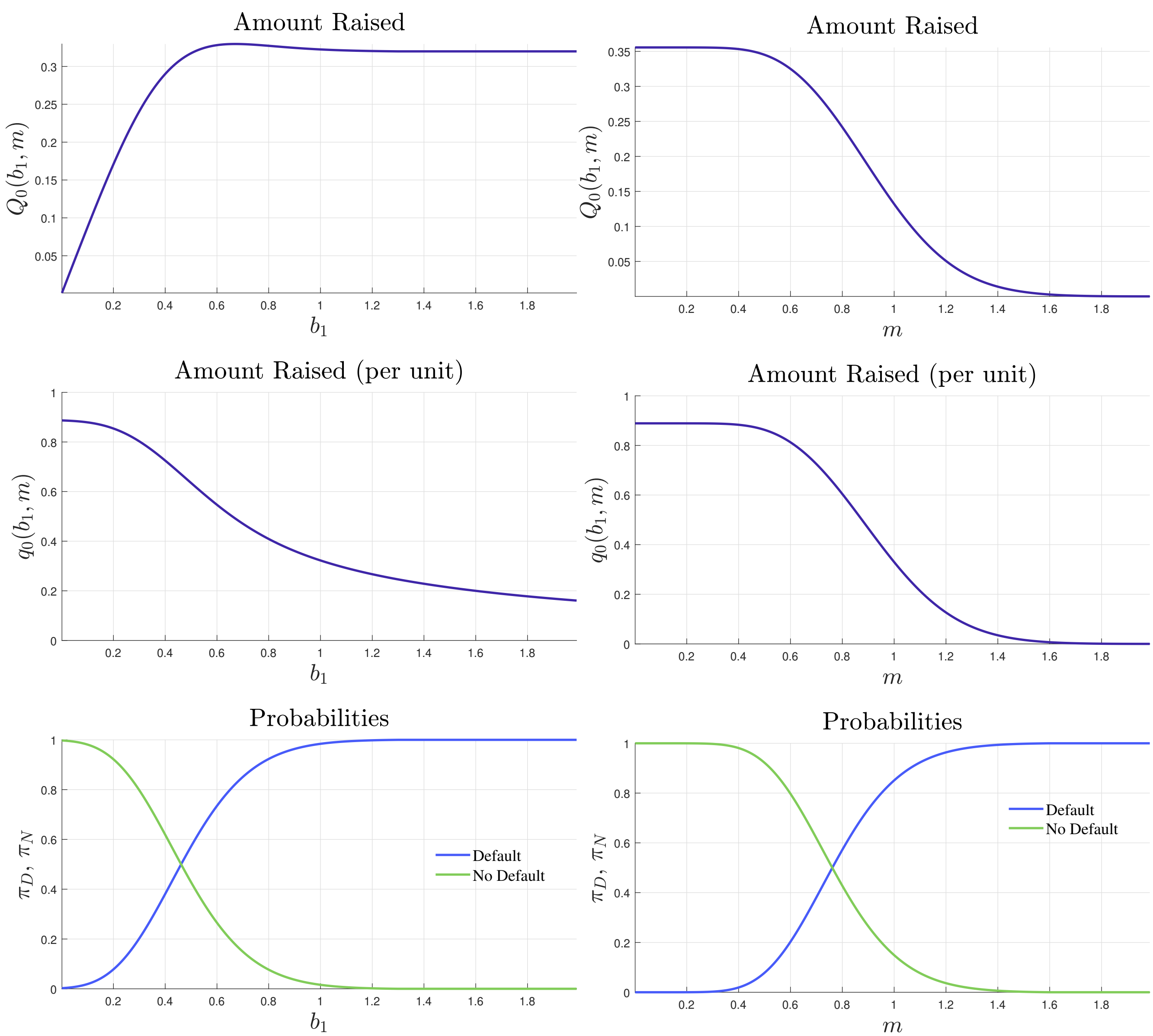

Figure 10: Credit Supply Schedule

Note: Figure 10 shows how the credit supply schedule varies with b_1 and exemption level m. Each row respectively shows total amount raised, amount raised per unit, and probabilities of default and no default. All figures have parameters $r^\ell = 1.04^3 - 1$, $\delta = 0.75$, $\mu = 0.2$, $\sigma = 0.18$, $\underline{s} = 0.2$, and $\overline{s} = 2$. The left plots set $m = 0.7$ and vary b_1. The right plots set $b_1 = 0.4$ and vary m.

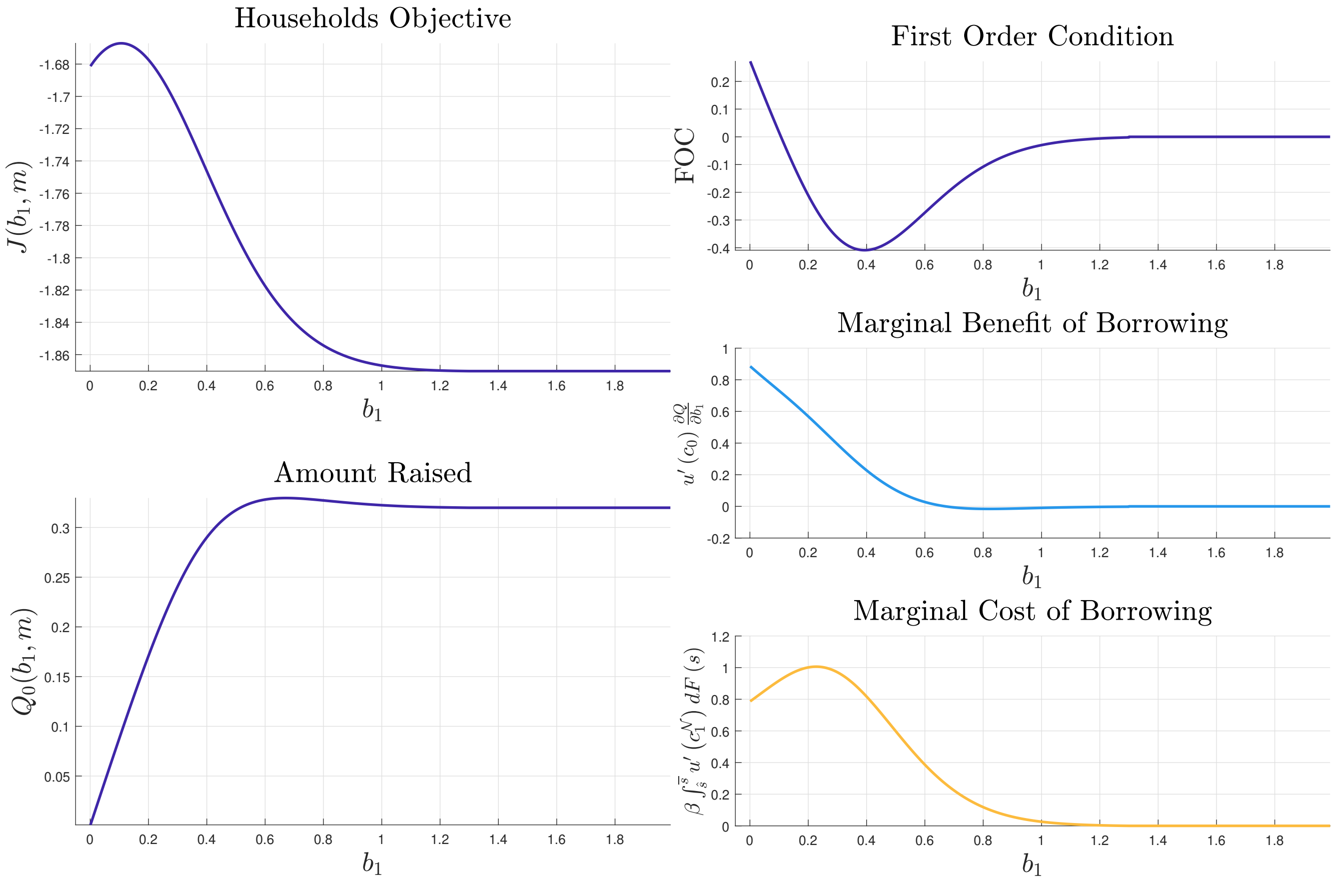

Figure 11: Households' Borrowing Problem

Note: Figure 11 illustrates the problem solved by households for a given exemption level. All figures have parameters $\beta = 0.92^3$, $r^\ell = 1.04^3 - 1$, $\gamma = 2$, $\delta = 0.75$, $n_0 = 1$, $\mu = 0.2$, $\sigma = 0.18$, $\underline{s} = 0.2$, and $\overline{s} = 2$. The exemption level is $m = 0.7$.

right plots illustrate the first order condition of the households' problem. The top right plot shows $\frac{dJ}{db_1} = u'(c_0)\frac{\partial Q_0}{\partial b_1} - \beta\int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s)$, while the bottom two right plot show the marginal benefit and the marginal cost of borrowing separately.

Figure 12 illustrates the shape taken by social welfare as a function of the exemption level m and the problem of selecting the optimal exemption. The top left plot shows social welfare as a function of the exemption level m. The bottom left plot illustrates households' equilibrium borrowing choice $b_1^\star(m)$ for different values of the exemption level m. The top right plot shows $\frac{dW}{dm}$ for different values of the exemption level, as calculated in Equation (13) in the text. The bottom right plot provides a decomposition in terms of the marginal cost, $u'(c_0)\frac{\partial Q_0}{\partial b_1}$, and benefit, $\beta\int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s)$, for different values of m. The bottom row of Figure 2 in the text also illustrates social welfare as a function of the exemption level. The top two plots in Figure 3 are analogous to the right plots in Figure 12. As described in the paper, total borrowing can be increasing or decreasing in m, so $\frac{db_1}{dm} \gtrless 0$. It is often the case in numerical simulations that $\frac{db_1}{dm}$ and $\frac{dW}{dm}$ have the same sign for large values of m. As illustrated in Figure 12, this is not a general conclusion.

H Additional Extensions

I include several extensions not explicitly studied in the body of the paper. Some of these extensions provide specific examples of the general framework analyzed in the paper, while others show how to

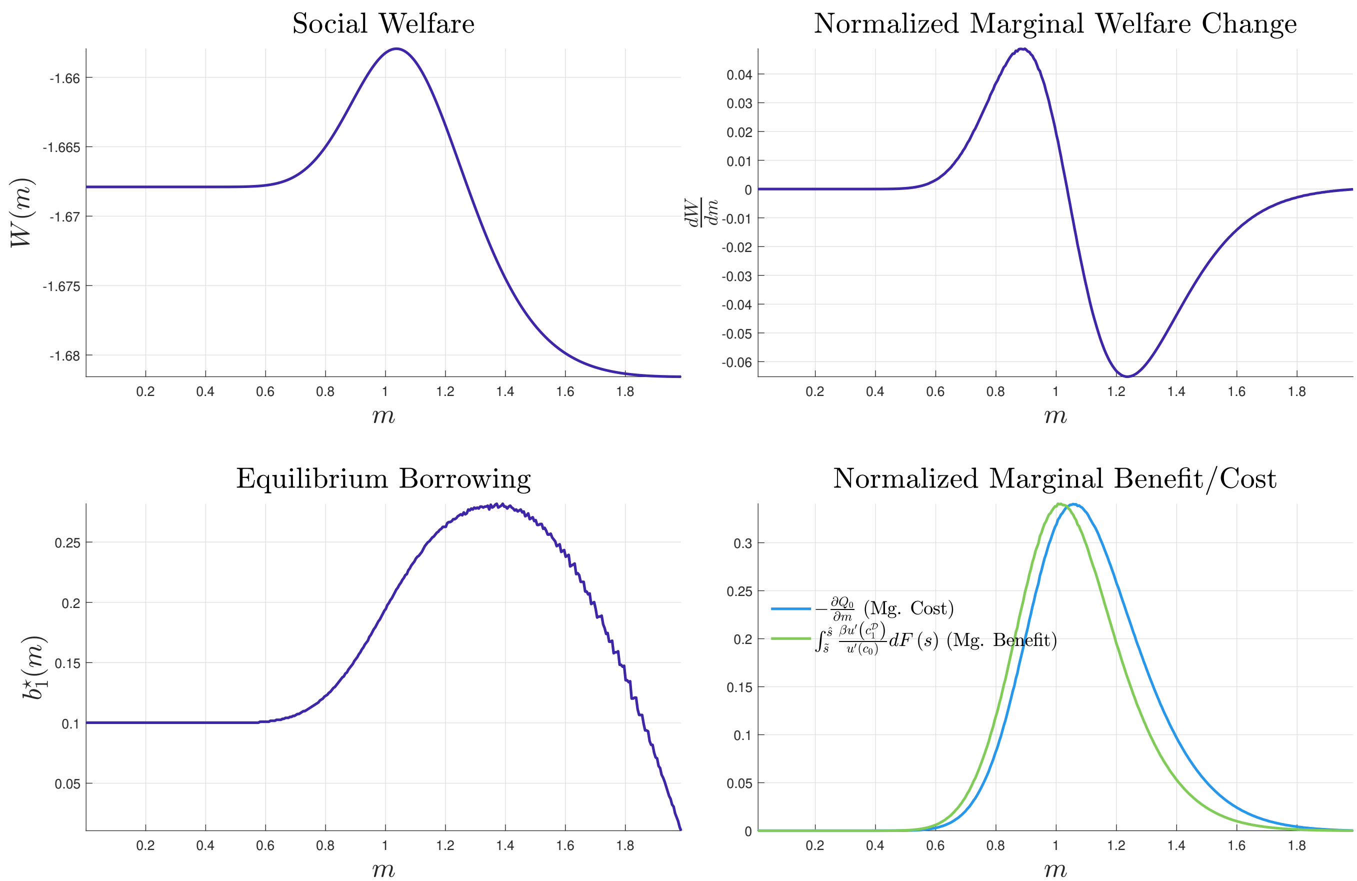

Figure 12: Social Welfare and Equilibrium Borrowing

Note: Figure 12 shows social welfare, its derivative with its decomposition, and equilibrium borrowing as a function of the exemption level. The left panel sets $m = 0.8$. All right figures have parameters $\beta = 0.92^3$, $r^\ell = 1.04^3 - 1$, $\gamma = 2$, $\delta = 0.75$, $n_0 = 1$, $\mu = 0.2$, $\sigma = 0.18$, $\underline{s} = 0.2$, and $\overline{s} = 2$. The optimal exemption in this simulation is $m^\star = 1.041$ and the equilibrium level of borrowing is $b_1^\star(m^\star) = 0.21$.

incorporate new features into the framework.

Long-Term Debt: The Perpetuity Case To simplify the analysis, and consistent with the empirical implementation that features only personal unsecured debt and auto loans, the paper exclusively considers one-period contracts. I now introduce a long-term bond in its most tractable form: a perpetuity. Formally, I consider a bond issued at a price $q(\cdot)$ that promises a stream of repayments $\alpha^{\tau-1}$, where $\alpha \geq 0$ and $\tau = 1, 2, \ldots$ denotes the number of periods after issuance. Therefore, in a given period, the amount of debt due for repayment is given by $d = s_1 + \alpha s_2 + \alpha^2 s_3 + \ldots$, where s_τ denotes the amount of debt issuance τ periods prior. The law of motion of debt is given by $d' = \alpha d + s$, where s denotes newly issued debt. See Uribe and Schmitt-Grohé (2017) for a textbook treatment of perpetuity debt in dynamic models of default.

Here I will consider a dynamic model in which households have an exogenously determined Markov endowment y and borrow in the form of a single perpetuity. We can express households' value function as

$$V(b, y; m) = \max\left\{V^{\mathcal{D}}(y; m), V^{\mathcal{N}}(b, y; m)\right\},$$

where $V^{\mathcal{D}}(\cdot)$ and $V^{\mathcal{N}}(\cdot)$ respectively denote continuation values after declaring bankruptcy and after repaying maturing liabilities. The continuation value of repaying liabilities is given by

$$V^{\mathcal{N}}(b, y; m) = \max u(c) + \beta \mathbb{E}\left[V(b', y'; m)\right],$$

where households' budget constraint is given by

$$c = y - b + q(b', y; m) \underbrace{(b' - \alpha b)}_{\text{Newly Issue Debt}}.$$

The continuation value of defaulting is given by

$$V^{\mathcal{D}}(b, y; m) = u(c) + \beta \mathbb{E}\left[V^{\mathcal{N}}(0, y'; m)\right],$$

where households' budget constraint after defaulting is given by

$$c = \mathcal{W}(y, m) = \min\{y, m\}.$$

This formulation assumes that households do not have access to credit markets after defaulting but that they immediately can borrow without penalty one period after defaulting. Both assumptions can be easily relaxed. As in the proof of Proposition 2, we can write $\frac{dV^{\mathcal{N}}}{dm}$ and $\frac{dV^{\mathcal{D}}}{dm}$ as follows

$$\frac{dV^{\mathcal{N}}}{dm}(b, y; m) = u'(c) \frac{\partial q(b', y; m)}{\partial m} \underbrace{(b' - \alpha b)}_{\text{Newly Issued Debt}} + \frac{d\mathbb{E}\left[V(b', y'; m)\right]}{dm}$$

$$\frac{dV^{\mathcal{D}}}{dm}(b, y; m) = u'(c) \frac{\partial \mathcal{W}}{\partial m} + \beta \mathbb{E}\left[\frac{dV^{\mathcal{N}}(0, y'; m)}{dm}\right],$$

where

$$\frac{d\mathbb{E}\left[V(b', y'; m)\right]}{dm} = \pi^{\mathcal{D}} \frac{d\mathbb{E}\left[V^{\mathcal{D}}(b', y'; m)\right]}{dm} + \pi^{\mathcal{N}} \frac{d\mathbb{E}\left[V^{\mathcal{N}}(b', y'; m)\right]}{dm}.$$

Solving forward this equations yields an expression like that in Equation (22). Comparing this characterization with that of the general model concludes that the only difference when considering a perpetuity model relative to a a model with one-period deb is that the flow marginal cost of changes in rates in the perpetuity model only must account for the amount of newly issued debt. It is possible to extend the results to explicitly consider alternative forms of long-term debt.

Credit Supply Determination under Pooling and Exclusion In this extension, I explicitly allow for pooling and exclusion of households from borrowing, by forcing lenders to submit price schedules that cannot be conditioned on individual households' characteristics. I show that allowing for unobservable heterogeneity does not modify the characterization of marginal welfare changes induced by changes in the exemption level, provided an equilibrium of the form considered here exists.[27] I focus on an environment in which lenders compete using a single pricing schedule and commit to it: once the price schedule is posted, lenders must lend the amount demanded at the posted rate. This is a natural choice to understand how changes in the exemption level affect the welfare of pooled and excluded borrowers.

Formally, I consider an extension of the baseline model in which households are unobservably heterogeneous in that they have different distributions of future endowments $F_i(\cdot)$. Although one could allow for different forms of heterogeneity, this is a natural case. I index households by i and assume that the distribution of households is given by $G(i)$, where $G(\cdot)$ denotes a well-behaved cdf. We can interpret $G(\cdot)$ as indexing a parameter (e.g., mean or variance) of the distribution of households' date 1 endowments.

Given Equation (34), which determines whether a given household finds it optimal to borrow or not, we can classify households into active or excluded borrowers for a given level of m. Formally, I respectively denote by $A(m)$ and $E(m)$ the sets of active and excluded borrowers, for a given m

$$\begin{aligned} A(m) &= \{i \mid b_{1i}(m) > 0\} \quad \text{Active borrowers} \\ E(m) &= \{i \mid b_{1i}(m) = 0\} \quad \text{Excluded borrowers,} \end{aligned}$$

It should be clear from Equation (34) that varying m causes borrowers to adjust their decisions both on the intensive and the extensive margin. Given the pooling nature of the equilibrium, I denote by $q_0(b_{1i}, m)$ the pricing schedule offered by lenders to all households, regardless of type. Given that households' default rule is identical ex-post (this can be relaxed), we can therefore express $q_0(b_{1i}, m) b_1$ as follows

$$q_0(b_{1i}, m) b_{1i} = \begin{cases} \frac{1}{1+r^{\ell}}, & b_{1i} \leq 0 \\ \int_{A(m)} \frac{\tilde{q}_{0j}(b_{1i}, m) b_{1i}}{\int_{A(m)} dG(j)} dG(j), & b_{1i} > 0, \end{cases}$$

where I define $\tilde{q}_{0j}(b_{1i}, m) b_{1i} = \frac{\delta \int_{\underline{s}}^{\hat{s}} \max\{n(s)-m\} dF_j(s) + b_{1i} \int_{\hat{s}}^{\overline{s}} dF_j(s)}{1+r^{\ell}}$ to represent the credit supply schedule that an individual of type i would receive if not pooled. In equilibrium, lenders offer a credit supply schedule that averages over the set of borrowers, taking into account that the level of m modifies the composition of borrowers in the economy. Given this price schedule, borrowers choose whether to borrow or not. Under regularity conditions, an equilibrium exists in which the credit supply schedules are consistent with the set of active borrowers, and vice versa.

Given that lenders profits do not vary with m, we can express the marginal welfare change for each household as in baseline model

$$\frac{dW_i}{dm} = \begin{cases} u'(c_{0i}) \frac{\partial q_0}{\partial m}(b_{1i}, m) b_1 + \beta \int_{\hat{s}}^{\hat{s}} u'\left(c_{1i}^{\mathcal{D}}(s)\right) dF_i(s), & \text{if active,} \quad b_{1i} > 0 \\ 0, & \text{if excluded,} \quad b_{1i} \leq 0, \end{cases}$$

[27] Once asymmetric information is introduced, many phenomena can arise. Analyzing all of them is outside of the scope of this paper. This example is meant to provide a natural environment in which the insights from the paper apply without modification.

with the caveat that $\frac{\partial q_0}{\partial m}(b_{1i}, m)$ now corresponds to the derivative of the pooled schedule. Importantly, because changing m does not affect excluded borrowers' welfare and because it is optimal for borrowers who move at the margin from being active to excluded to do so since households' indirect utility W_i is continuous at $b_{1i} = 0$, changes in social welfare only need to account for how individual welfare changes for the set of active borrowers.

There is no difference between observable and unobservable heterogeneity for the purpose of understanding the welfare implications of changing the level of the bankruptcy exemption. With unobservable heterogeneity, varying m changes the composition of borrowers not only directly but also through endogenous changes in the pricing schedules faced by borrowers in equilibrium, which creates feedback and modifies borrowers choices on the extensive and intensive margins. However, all the information needed to assess whether a change in m increases or decreases welfare is embedded in the sufficient statistics identified in this paper, in particular, the sensitivity of credit supply schedules.

Epstein-Zin Preferences The framework developed in the paper can accommodate non-expected utility preferences. The objective function of households with Epstein-Zin utility corresponds to

$$v_0 = \left[\left(1-\hat{\beta}\right) c_0^{1-\frac{1}{\psi}} + \hat{\beta}\left(\mathbb{E}\left[c_1^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}.$$

In that case, households' optimal borrowing choice can be characterized as the solution to

$$W(m) = \max_{b_1}\left[\left(1-\hat{\beta}\right) c_0^{1-\frac{1}{\psi}} + \hat{\beta}\tilde{V}(b_1, m)\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

where $c_0 = n_0 + Q_0(b_1, m)$ and $\tilde{V}(b_1, m)$ is given by

$$\tilde{V}(b_1, m) = \left(\int_{\underline{s}}^{\hat{s}} \left(c_1^{\mathcal{D}}(s)\right)^{1-\gamma} dF(s) + \int_{\hat{s}}^{\overline{s}} \left(c_1^{\mathcal{N}}(s)\right)^{1-\gamma} dF(s)\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

The welfare change induced by a marginal change in the bankruptcy exemption m (the counterpart of Proposition 1 in the text) is given by

$$\frac{dW}{dm} = v_0^{\frac{1}{\psi}}\left[\left(1-\hat{\beta}\right)(c_0)^{-\frac{1}{\psi}}\frac{\partial q_0}{\partial m} b_1 + \hat{\beta} Z^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} \int_{\tilde{s}}^{\hat{s}} \left(c_1^{\mathcal{D}}(s)\right)^{-\gamma} dF(s)\right].$$

After normalizing by $v_0^{\frac{1}{\psi}}\left(1-\hat{\beta}\right)(c_0)^{-\frac{1}{\psi}}$, it is evident that given the other sufficient statistics, changing preferences only affects the expression for the marginal welfare change induced by a change in the exemption level through its effect on how value marginal gains and losses over time/across states. As expected, when $\gamma = \frac{1}{\psi}$, we recover the results from Proposition 1.

Price-Taking Households Instead of assuming that households internalize the impact of their borrowing choices on the interest rate they get charged, one could assume that borrowers act as interest rate takers, at least within a given range. In a price-taking equilibrium, households submit a loan demand, given interest rates, and lenders submit a loan supply schedule, whose intersection determines a standard competitive equilibrium.

The problem of price taking households has two local optima, an interior optimum, characterized by Equation (37), and a second optimum that entails borrowing as much as possible. Although this may create non-existence problems, by judiciously setting a non-binding upper bound on credit, as discussed by Eaton and Gersovitz (1981), it is possible to select the interior solution. See also Athreya (2002).

In this case, households date 0 borrowing choices are characterized by the following Euler equation

$$u'(c_0)\, q_0 = \beta \int_{\hat{s}}^{\overline{s}} u'\left(c_1^{\mathcal{N}}(s)\right) dF(s). \tag{37}$$

This condition differs from Equation (11) in the text since, since households perceive that increasing b_1 at the margin allows them to raise a fixed q_0 dollars at date 0.

The welfare change induced by a marginal change in the bankruptcy exemption m (the counterpart of Proposition 1 in the text) is given by

$$\frac{dW}{dm} = u'(c_0)\, \frac{dq_0}{dm} b_1 + \beta \int_{\tilde{s}}^{\hat{s}} u'\left(c_1^{\mathcal{D}}(s)\right) dF(s),$$

where the sensitivity $\frac{dq_0}{dm}$ accounts for all equilibrium effects.

There is an important difference between the results of the baseline model, in which borrowers internalize the effect of borrowing on interest rates, and the results in which borrowers are price takers. The relevant variable to measure the sensitivity of interest rates to exemption changes is now the full equilibrium response of interest rates, which also includes the change in interest rates induced by equilibrium changes in total borrowing. Because price taking borrowers fail to internalize that higher borrowing increases interest rates, this effect needs to be accounted for when measuring welfare.

Endogenous Contract Response Following the general equilibrium tradition, this paper has taken the payoff structure and number of available contracts as part of the environment. This is consistent with the observation that debt contracts are widespread in high and low exemption regions. However, this approach implies that the shape of contracts used is invariant to the level of exemptions. I now show that even if the shape of contracts endogenously responds to the exemptions level, the characterization of the marginal welfare change remains valid.

In this case, households solve the following problem

$$W(m) = \max_{b_1, \{\rho(s,m)\}} u(c_0) + \beta \mathbb{E}\left[u(c_1(s))\right],$$

subject to a date 0 budget constraint $c_0 = n_0 + Q_0(b_1, m; \{\rho(s,m)\})$ and the following date 1 budget constraints

$$\begin{aligned} c_1^{\mathcal{N}}(s) &= n(s) - \rho(s,m)\, b_1 \\ c_1^{\mathcal{D}}(s) &= \min\{n(s), m\}. \end{aligned}$$

The function $\rho(s,m)$ allows for a full nonlinear contract shape, in which repayment can be made contingent on s, and can potentially vary with the level of exemptions. If unrestricted, households would select a contract that recovers the complete markets first-best. I will assume that adjusting the contract shape along a given dimensions s incurs the cost $\zeta(\rho(s,m))$. A formal proof of the results would involve using optimal control. Instead, I provide a heuristic argument, in which I use $\{\rho(s,m)\}$ to denote the full contract and $\rho(s,m)$ to denote the value of the contract for a given value of s.

The pricing schedule offered by lenders, now also a function of $\rho(s,m)$ is given by

$$Q_0(b_1, m; \{\rho(s,m)\}) = \frac{\delta \int_{\mathcal{D}} \max\{n(s) - m, 0\}\, dF(s) + b_1 \int_{\mathcal{N}} \rho(s,m)\, dF(s)}{1 + r^{\ell}}.$$

Borrowers optimally conditions for credit and the contract shape are given by

$$u'(c_0)\frac{\partial Q_0}{\partial b_1} = \beta\int_{\hat{s}}^{\overline{s}}\rho(s,m)\,u'\left(c_1^{\mathcal{N}}(s)\right)dF(s)$$
$$u'(c_0)\frac{\partial Q_0}{\partial \rho(s,m)} = \beta\int_{\hat{s}}^{\overline{s}}u'\left(c_1^{\mathcal{N}}(s)\right)b_1 dF(s) + \zeta'(\rho(s,m)),\ \forall s.$$

In this case, the welfare change induced by a marginal change in the bankruptcy exemption m (the counterpart of Proposition 1 in the text) is given by

$$\frac{dW}{dm} = u'(c_0)\frac{\partial q_0}{\partial m}b_1 + \beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\right)dF(s),$$

which corresponds to Equation (13) in the text. Although now a change in the exemption level can potentially modify the terms of the contract, because households choose the shape of $\rho(s,m)$ optimally, the welfare impact of the change remains unchanged.

Non-Pecuniary Utility Bankruptcy Cost To explicitly account for state-dependent utility in case of bankruptcy (capturing stigma or other private costs associated with bankruptcy) I assume households' utility in bankruptcy is given by $u(\phi c)$, where $\phi \in [0,1)$. In this case, households' optimal bankruptcy decision is given by:

$$\begin{aligned}&\text{if } n(s) - b_1 > \phi m, && \text{Default}\\ &\text{if } n(s) - b_1 \leq \phi m, && \text{No Default,}\end{aligned}$$

which defines new thresholds $\hat{s} = n^{-1}(\phi m + b_1)$ and $\tilde{s} = n^{-1}(m)$. In this case, the welfare change induced by a marginal change in the bankruptcy exemption m (the counterpart of Proposition 1 in the text) is given by

$$\frac{dW}{dm} = u'(c_0)\frac{\partial q_0}{\partial m}b_1 + \beta\int_{\tilde{s}}^{\hat{s}}\phi u'\left(\phi c_1^{\mathcal{D}}\right)dF(s).$$

An in Epstein-Zin case, given the other sufficient statistics, changing preferences only affects the expression for the marginal welfare change induced by a change in the exemption level through its effect on how value marginal gains and losses over time/across states.

General Lenders' Pricing Kernel It is possible to allow for lenders to have different pricing kernel than households. Formally, in that case the credit supply schedule can be written as

$$Q_0(b_1,m) = \frac{\delta\int_{\mathcal{D}}\max\{n(s)-m,0\}\,dH(s) + b_1\int_{\mathcal{N}}dH(s)}{1+r^{\ell}},$$

where $H(\cdot)$ is a well-behaved cdf (an absolutely continuous change of measure with respect to $F(\cdot)$). The characterization in Proposition 1 remains unchanged in this case.

Aggregate Demand Effects I now illustrate how accounting for aggregate demand externalities affects the characterization of the marginal welfare change.[28] For simplicity, I only introducing production at $t = 0$, assuming that output is generated by a constant returns to scale production function $y_0 = z_0 l_0$, where z_0 is a constant productivity parameter and l denotes the amount labor employed. Households

[28]There is scope to further analyze the interaction of bankruptcy exemptions and aggregate demand in a quantitative model.

date 0 utility is given by $u\left(c_0,l_0\right)$. At the first-best, wages equal the marginal product of labor, and the condition $w_0=z_0$ must hold. Instead, I assume that $w_0=\left(1+\tau_0\right)z_0$, where $\tau_0=\frac{w_0}{z_0}-1$ denotes the labor wedge in this economy. This wedge, which maps one-to-one to an output gap, naturally emerges under nominal rigidities and imperfect monetary policy – see, for instance, Farhi and Werning (2016).

In this case, households maximize

$$\max u\left(c_0,l_0\right)+\beta\mathbb{E}\left[\max\left\{u\left(c_1^{\mathcal{D}}\left(s\right)\right),u\left(c_1^{\mathcal{N}}\left(s\right)\right)\right\}\right],$$

subject to a date 0 budget constraint $c_0=n_0+w_0l_0^{\mathcal{N}}+\pi_0+Q_0\left(b_1,m\right)$ and date 1 budget constraints defined in Equations (4) and (5). Households optimality conditions are given by

$$\frac{\partial u}{\partial c}\frac{\partial Q_0}{\partial b_1}-\beta\int_{\mathcal{N}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)dF\left(s\right)=0$$
$$w_0\frac{\partial u}{\partial c}\left(c_0,l_0\right)+\frac{\partial u}{\partial l}\left(c_0,l_0\right)=0.$$

Social welfare is given by the indirect utility to the problem solved by households, with the exemption that $c_0=n_0+z_0l_0+Q_0\left(b_1,m\right)$. Assumption that $w_0=\left(1+\tau_0\right)z_0$

$$\begin{aligned}\frac{dW}{dm}&=\frac{\partial u}{\partial c}\left(c_0,l_0\right)\frac{dc_0}{dm}+\frac{\partial u}{\partial l}\left(c_0,l_0\right)\frac{dl_0}{dm}-\beta\int_{\hat{s}}^{\overline{s}}u'\left(n\left(s\right)-b_1\right)dF\left(s\right)\frac{db_1}{dm}+\beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\right)dF\left(s\right)\\&=\frac{\partial u}{\partial c}\left(c_0,l_0\right)\frac{\partial Q_0\left(b_1,m\right)}{\partial m}+\beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\right)dF\left(s\right)+\left[\frac{\partial u}{\partial c}\left(c_0,l_0\right)z_0+\frac{\partial u}{\partial l}\left(c_0,l_0\right)\right]\frac{dl_0}{dm}\\&=\frac{\partial u}{\partial c}\left(c_0,l_0\right)\frac{\partial Q_0\left(b_1,m\right)}{\partial m}+\beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\right)dF\left(s\right)+\frac{\partial u}{\partial c}\left(c_0,l_0\right)\left(1-\frac{w_0}{z_0}\right)\frac{dy_0}{dm},\end{aligned}$$

where we use households consumption-leisure optimal choice, and the fact that $\frac{dc_0}{dm}=z_0\frac{dl_0}{dm}+\frac{\partial Q_0(b_1,m)}{\partial m}+\frac{\partial Q_0(b_1,m)}{\partial b_1}\frac{db_1}{dm}$ and $\frac{dl_0}{dm}=\frac{1}{z_0}\frac{dy_0}{dm}$. We can therefore express the normalized welfare change as follows

$$\frac{\frac{dW}{dm}}{\frac{\partial u}{\partial c}\left(c_0,l_0\right)}=\frac{\partial q_0\left(b_1,m\right)}{\partial m}b_1+\beta\int_{\tilde{s}}^{\hat{s}}u'\left(c_1^{\mathcal{D}}\right)dF\left(s\right)-\underbrace{\tau_0\frac{dy_0}{dm}}_{\substack{\text{Agg. Demand}\\\text{Effect}}}.$$

The new aggregate demand term shows that if increasing exemptions boosts aggregate demand when there is a demand shortfall (and moderates aggregate demand when the economy is overstimulated), there is an additional rationale to increase exemption levels motivated by managing macroeconomic fluctuations.