Discussion

A Dynamic Theory of Multiple Borrowing

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Summary

- Facts
  - Borrowers often borrow from multiple lenders sequentially
  - Many models assume that borrowers borrow from a single lender
- This paper explores the role of sequential lending
  - from multiple borrowers
  - without commitment
- Interesting and relevant question

Summary

- Facts
  - Borrowers often borrow from multiple lenders sequentially
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- This paper explores the role of sequential lending
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  - without commitment
- Interesting and relevant question
- Main takeaways
  - Early lenders internalize that borrowers will borrow from others
  - More productive projects may end up getting less financing
  - Having more (sequential) lenders decreases welfare
    - Second-best result
- Mechanism
  - Late lenders do not internalize the impact of new debt on early lenders repayment

Roadmap of my discussion

1. Review of the basic argument
2. Review of the dynamic argument
3. Comments and thoughts

Static environment

- Risk-neutral borrowers solve (small notation changes)

$$\max_{D,K} zK - \underbrace{\int_{\underline{c}}^{D} cf(c)\,dc}_{\text{Default}} - \underbrace{D\int_{D}^{\bar{c}} f(c)\,dc}_{\text{Repayment}}$$

- Debt contract

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- Debt contract
- Risk-neutral lenders price debt as (credit surface)

$$K = D\int_{D}^{\bar{c}} f(c)\,dc \Rightarrow \boxed{K(D)}$$

  - $K(D)$ is a Laffer curve
  - $\lim_{D\to 0} K(D) = 0$ and $\lim_{D\to \bar{c}} K(D) = 0$

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- Remark: default is driven here by exogenous cost, not by low realizations of output
  - More natural to write: (definitely more parsimonious)

$$\max_{D,K} \int \max\{z(s)K - D\} f(s)\,ds$$

Static solution

- When lenders have all bargaining power

$$\max_{D} zK(D) - \int_{\underline{c}}^{D} cf(c)\,dc - D\int_{D}^{\bar{c}} f(c)\,dc$$

- Solution (if interior):

$$\underbrace{zK'(D)}_{\text{Mg. Benefit}} - \underbrace{\int_{D}^{\bar{c}} f(c)\,dc}_{\text{Mg. Cost}} = 0$$

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- Euler equation
  - Mg. Benefit of borrowing: higher investment
  - Mg. Cost of borrowing: repaying the debt in no default states
- Solution on upward-sloping side of the Laffer curve $K'\left(D\right) > 0$

Commitment Problem

- After borrowing D optimally, borrower meets a new lender
- New objective

$$\max_{D_n} U^B, \quad \text{where}$$

$$U^B = z\left(K\left(D^\star\right) + K_n\left(D_n\right)\right) - \int_{\underline{c}}^{D^\star + D_n} cf\left(c\right) dc - \left(D^\star + D_n\right) \int_{D^\star + D_n}^{\overline{c}} f\left(c\right) dc$$

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- Lenders pricing

$$K_n\left(D_n\right) = D_n \int_{D + D_n}^{\overline{c}} f\left(c\right)dc$$

- Compare to $K\left(D\right) = D\int_{D}^{\overline{c}} f\left(c\right)dc$
- Remark: because recovery rate after default $= 0 \Rightarrow$ No role for seniority (binary payoff)

Overborrowing Argument (Bizer/DeMarzo 92, Theorem 2)

- Compare two perturbations around the originally optimal D
  - Borrow from the new lender vs. borrow from the original lender

$$\left.\frac{dU^B}{dD_n}\right|_{D_n=0} = z\, K_n'(D_n)\big|_{D_n=0} - \int_D^{\bar{c}} f(c)\,dc > 0$$

$$\left.\frac{dU^B}{dD}\right|_{D_n=0} = zK'(D) - \int_D^{\bar{c}} f(c)\,dc = 0$$

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$$\left.\frac{dU^B}{dD}\right|_{D_n=0} = zK'\left(D\right) - \int_D^{\bar{c}} f\left(c\right) dc = 0$$

- Marginal cost are the same (!)
- Marginal benefits are different $\left(K_n'\left(D_n\right)\big|_{D_n=0} > K'\left(D\right)\right)$

$$K'\left(D\right) = \int_D^{\bar{c}} f\left(c\right) dc - Df\left(d\right)$$
$$K_n'\left(D_n\right) = \int_{D+D_n}^{\bar{c}} f\left(c\right) dc - D_n f\left(D+D_n\right)$$

- So $K_n'\left(D_n\right)\big|_{D_n=0} = \int_D^{\bar{c}} f\left(c\right) dc$ (the last term drops)

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- So $K_n'\left(D_n\right)\big|_{D_n=0} = \int_D^{\bar{c}} f\left(c\right) dc$ (the last term drops)
- **Key Idea**: original lender internalizes that higher debt makes default more likely, lowers the repayment on existing debt
- Next step: early lenders should rationally expect this

Dynamic Environment

- Borrowers' (back notation in the paper)

$$V(D) = \max_{D'} \left\{ zK + (1-q)\left(-\mathbb{E}\left[\min\left(D', c\right)\right]\right) + qV\left(D'\right) \right\}$$

subject to

$$K = p^{\star}\left(D'\right)\left(D' - D\right)$$

- Lenders profit is $\mathbb{E}\left[\Pi^{i}_{\text{lenders}}\right] = p\left(D'\right) d_i + \left(1 - p\left(D'\right)\right) \cdot 0$

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- Paper looks for Stationary Markov Linear Equilibrium
  - $p^{\star}(\cdot)$ and $D^{\star}(D)$
  - Closed-form solution: quadratic value function (clever!)
- Several simplifications to preserve tractability
  - Repayment does not depend on investment K
  - Risk neutrality
  - Short-term debt
  - Ad-hoc default cost $c \sim U[0,1]$
- Paper shows that stationary solution is the limit SPE with many periods

Main results

1. More lenders (higher q) $\Rightarrow$ Worse allocations
   - More borrowing, less investment, more default, **lower welfare**
2. Increase in z (better opportunities) $\Rightarrow$ Worse commitment problem
   - More borrowing, potentially lower investment (debt is so high that dilution is terrible), but **welfare goes up**
   - Remark: in this model, higher z means higher desire to borrow mechanically. Unclear whether this generalizes to more instruments or random z

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- Extensions:
  1. Pledgeability: debt, investment, welfare go up, but higher ability to borrow makes commitment problem worse
  2. Lenders with limited funds: ambiguous effects
  3. Concave returns: limits commitment problem
  4. Nash bargaining
- Policy responses: caps and taxes

Comments/Thoughts

1. It would be useful to consider commitment options
   - Lack-of-commitment is the right assumption
   - However, we see ex-ante behavior adopted to alleviate ex-post lack of commitment
     (a) Covenants (can eliminate problem)
     (b) Seniority (can mitigate problem)
     (c) Alternative contracts besides debt (can mitigate problem)
   - Coase theorem: we do see people internalizing the externalities
   - Commitment vs. flexibility (AWA, HY, others)
   - Corporate vs. households vs. sovereign

Comments/Thoughts

2. Which additional insights we get from the infinite horizon relative to three period model?
   - Tractability is nice, but restrictive
     - Ad-hoc default decision
     - No recovery after default
     - Uniform (!) distribution of default costs
   - Would like to see those assumptions relaxed
     - It should be doable in 3 periods
3. Scope for more quantitative work?
   - Similar effects explored quantitatively in sovereign default
   - Not that much in corporate finance