Discussion
Private Renegotiations and Government Interventions in Debt Chains

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This paper

- **Motivation**: Borrowers and lenders are interconnected
  - Often via *debt/credit chains*
  - Agent 1 lends to 2, who lends to 3, who lends to 4, etc.
  - Agents are borrowers and lenders at the same time
  - How to think of renegotiation in these environments?

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  - Agents are borrowers and lenders at the same time
  - How to think of renegotiation in these environments?
- **This paper:** Renegotiation in debt chains
  - Default DWL's are inefficient
  - Can default-free renegotiation be an equilibrium?
  - Policy interventions: subsidies vs debt reductions
  - Early vs late renegotiation

High-Level Summary

- Debt chain

$$\underbrace{1}_{\text{lender}} \longrightarrow 2 \longrightarrow \cdots \longrightarrow j-1 \to j \longrightarrow \cdots \longrightarrow N-1 \longrightarrow \underbrace{N}_{\text{borrower}}$$

  - Debt repayments flow from N towards 1
  - N is "downstream borrower"
  - Risk-neutral agents

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- Each lender chooses how much debt to forgive
  - Lenders have full bargaining power (bilateral relation)
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  - **Remark**: wlog, since since they can demand the promised amount
- Only inefficiency are default DWL's
  - First-best: wipe out debt or subsidize everyone

Main Results

1. Characterization of default-free renegotiation
2. Subsidies to "downstream" borrowers are more effective
   - Directly (as expected)
   - Strategically (upstream lenders have stronger incentives to renegotiate)
3. Improving lenders' conditions (perhaps via debt reductions) changes incentives to renegotiate
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- Several extensions
  - Borrower-specific default costs, asset interdependence, etc.

Outline of Discussion

1. Illustration of results
   - Basic tradeoff
   - Debt chain
2. Comments/Thoughts

Illustration of the results: Basic tradeoff

- Two dates; one borrower + one lender: $N = 2$
- Borrower's default decision (final date)
  - If $v - d < 0 \Rightarrow$ Default
  - If $v - d \geq 0 \Rightarrow$ Repay

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- Lender's profit/utility: Lender's "Laffer curve"
  - Credit supply/credit surface

$$\Pi^{\ell}(d) = \overbrace{(1-\rho)\int_{\underline{v}}^{d} v dF(v)}^{\text{Default}} + \overbrace{d\int_{d}^{\overline{v}} dF(v)}^{\text{Repayment}}$$

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- FOC:

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- Original debt $\overline{d}$ is irrelevant (as long as $d \leq \overline{d}$)
- Mg. Cost is social/Mg. Benefit is private

Illustration of the results: Basic tradeoff

- Interior optimum:

$$\frac{\partial \Pi^{\ell}(d)}{\partial d} = 0 \Rightarrow \frac{1 - F(d^{\star})}{f(d^{\star})} = \rho d^{\star} \Rightarrow \boxed{d^{\star} = \frac{1}{\rho} \frac{1}{\frac{f(d^{\star})}{1-F(d^{\star})}}}$$

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- Default-free renegotiation:

$$\frac{\partial \Pi^{\ell}(d)}{\partial d} \leq 0 \Rightarrow \frac{1 - F(d)}{f(d)} \leq \rho d \Rightarrow \boxed{d^{\star} = \underline{v}}$$

  - Equation (17) in the paper: applies to $j = 1$

Illustration of the results: Chain

- Lender's profit/utility in the chain:

$$\Pi^{\ell}_{j-1}\left(d_j\right) =$$

$$= \int_{\underline{v}_{j-1}}^{\overline{v}_{j-1}} \int_{\underline{v}_j}^{\overline{v}_j} \max \left\{ \overbrace{v_{j-1} - d_{j-1} + \underbrace{(1-\rho)\left(v_j + d_{j+1}\right)\mathbf{1}_{\mathcal{D}}}_{j \text{ defaults}} + \underbrace{d_j \mathbf{1}_{\mathcal{N}}}_{j \text{ repays}}}^{j-1 \text{ repays}}, \overbrace{0}^{\substack{j-1 \\ \text{defaults}}} \right\} dF\left(v_j, v_{j-1}\right)$$

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- Again, we can look at $\frac{\partial \Pi^{\ell}_{j-1}\left(d_j^{\star}\right)}{\partial d_j} = 0$ to find best response $d_j^{\star}$
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  - The optimal $d^\star_j\left(\cdot\right)$ is a function of d_{j+1} and d_{j-1}!

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- **Comment:** it'd be nice to provide comparative statics:

$$\frac{\partial d^{\star}_j}{\partial d_{j+1}} \quad \text{and} \quad \frac{\partial d^{\star}_j}{\partial d_{j-1}}$$

  - Similar when introducing subsidies: $\frac{\partial d^{\star}_j}{\partial s_j}, \frac{\partial d^{\star}_j}{\partial s_{j+1}}, \frac{\partial d^{\star}_j}{\partial s_{j-1}}$
  - Also comparative statics on ρ, $F(v)$, $\underline{v}$, $\overline{v}$

Summary of the forces

1. Lenders like to be paid as much as possible
2. Lenders want to forgive debt to reduce default DWL's
3. Lenders only care about future payments when they are not defaulting

- These are all very general forces
- Present in any renegotiation environment

Comments/Thoughts

1. Paper focuses on **conditions under which default-free (efficient) equilibrium exists**
   - One could think of environments in which default is unavoidable
     - For example: $\underline{v}_j \leq 0$?
   - Worst case scenarios $\underline{v}_j$ are critical
   - The paper already has a theory of renegotiation with default
   - It can be developed further

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2. Why not compare decentralized renegotiation with efficient and **constrained efficient solutions**?
   - Constrained efficiency: the planner internalizes impact on renegotiations
   - Terms $\frac{\partial d_{j+1}}{\partial d_j} \neq 0$ or $\frac{\partial d_{j-1}}{\partial d_j} \neq 0$ in the FOC?
     - Analogy: Stackelberg vs Cournot

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        - Analogy: Stackelberg vs Cournot
3. Focusing on $N = 3$ agents **should be sufficient** for many of the insights
    - Three equations, three unknowns

Comments/Thoughts

4. Role of **private/imperfect information**
    - There is no signaling/filtering/learning in the model
    - It is important that lenders (who determine the renegotiation conditions) default with positive probability
    - Maybe *uncertainty* is a better term?

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   - Harder question
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5. **Richer forms of interconnection**
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   - Chain is a particular network structure
6. **Ex-ante stage**
   - What triggers renegotiation?
   - What if agents anticipate this?